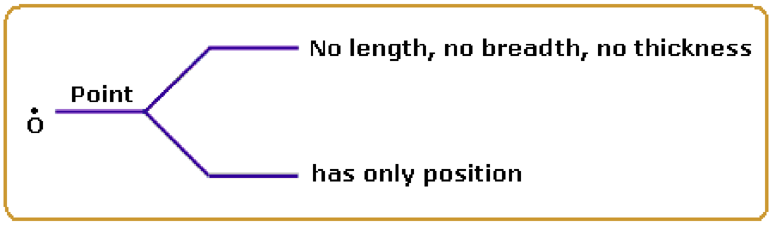
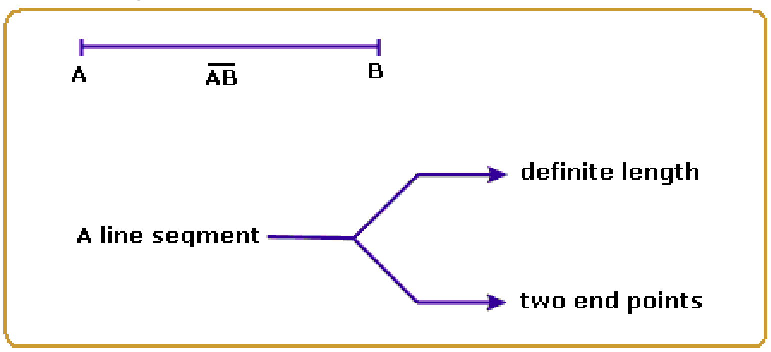
Lines and Angles

Geometrical Concepts

Point
It is an exact location. It is a fine dot which has neither length nor breadth nor thickness but has position i.e., it has no magnitude. It is denoted by capital letters A, B, C, O etc.



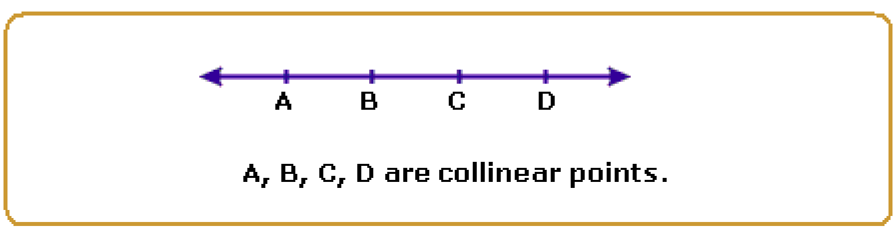
Line Segment
The straight path joining two points A and B is called a line segment $\overline{AB}$. It has end points and a definite length. (no breadth or thickness)



Ray
A line segment which can be extended in only one direction is called a ray.

Line
When a line segment is extended indefinitely in both directions it forms a line.

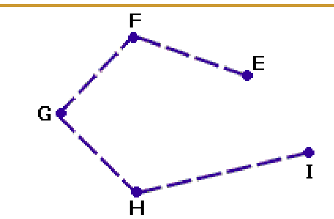
Collinear Points
If two or more points lie on the same line, then they are called collinear points.



Non-collinear Points

Points which do not lie on the same line are called non-collinear points.

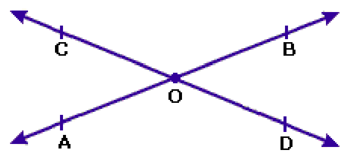
Example: E, F, G, H, I



E, F, G, H, I are non-collinear points.

Intersecting Lines

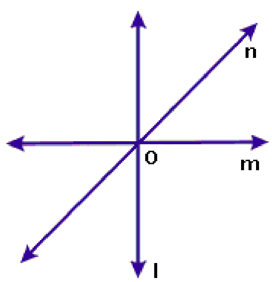
Two lines having a common point are called intersecting lines. The common point is known as the point of intersection.



O is the point of intersection

Concurrent Lines

If two or more lines intersect at the same point, then they are known as concurrent lines.



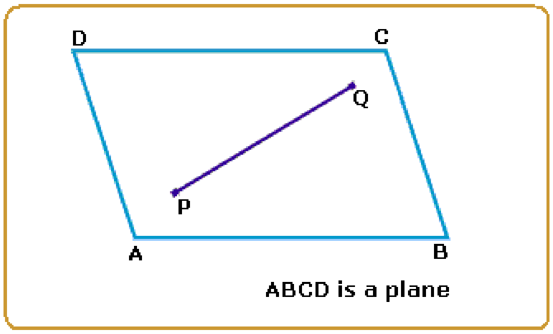
l, m, n are concurrent lines

Plane

A plane is a surface such that every point of the line joining any two points on it, lies on it.

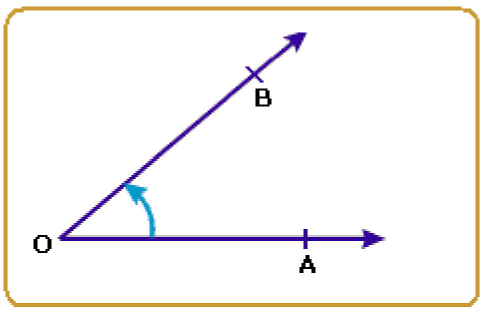
Example:

Surface of a smooth wall, surface of a paper.



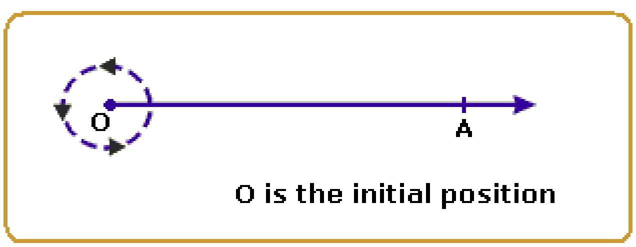
Angles

When two straight lines meet at a point they form an angle.



It is represented as ∠AOB or $A\hat{O}B$.

- $\overline{OA}$ and $\overline{OB}$ are called the arms of ∠AOB.
- The point at which the arms meet (O) is known as the vertex of the angle.
- The amount of turning from one arm (OA) to other (OB) is called the measure of the angle (∠AOB) and written as m∠AOB.
- An angle is measured in degrees, minutes and seconds.
- If a ray rotates about the starting initial position, in anticlockwise direction, comes back to its original position after 1 complete revolution then it has rotated through 360^{o}.



⇒1 complete rotation is divided into 360 equal parts. Each part is 1^{o}.

Each part (1^{o}) is divided into 60 equal parts, each part measuring one minute, written as 1'.

1' is divided into 60 equal parts, each part measuring 1 second, written as 1".

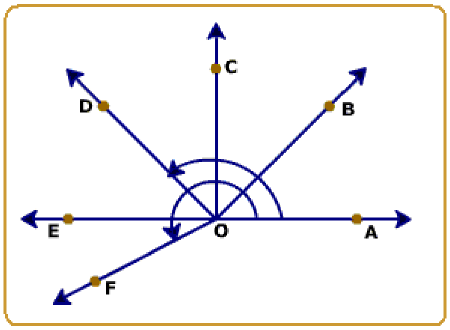
Degrees -----> minutes --------> seconds

$1^{o} = 60'$

$1' = 60''$

Recall that the union of two rays forms an angle.

In the figure, observe the different types of angles:

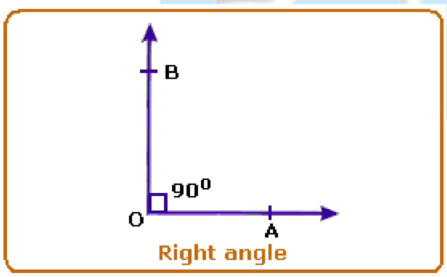


- $A\hat{O}B$ is an acute angle ($0^0 < A\hat{O}B < 90^0$)
- $A\hat{O}C$ is a right angle (an angle equal to 90^0)
- $A\hat{O}D$ is an obtuse angle ($90^0 < A\hat{O}D < 180^0$)
- $A\hat{O}E$ is a straight angle (an angle equal to 180^0)
- $A\hat{O}F$ (measured in anticlock wise direction) is a reflex angle

  ($180^0 < A\hat{O}F < 360^0$)

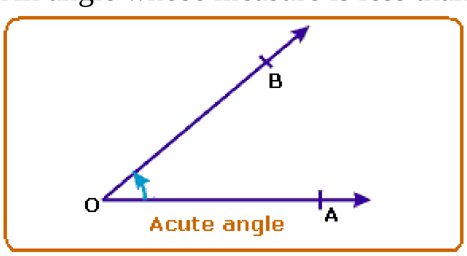
Right Angle

An angle whose measure is 90^o is called a right angle.



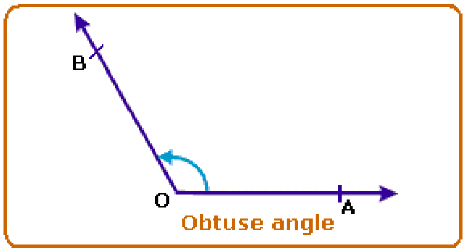
Acute Angle

An angle whose measure is less than one right angle (i.e., less than 90^o), is called an acute angle.



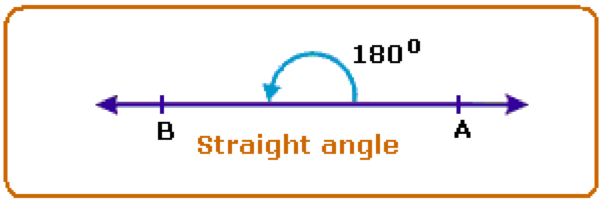
Obtuse Angle

An angle whose measure is more than one right angle and less than two right angles (i.e., less than 180^o and more than 90^o) is called an obtuse angle.



Straight Angle

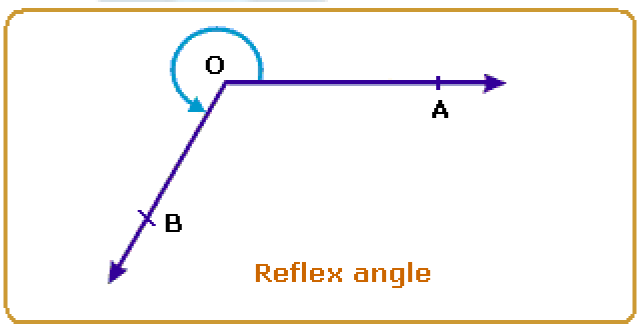
An angle whose measure is 180^o is called a straight angle.



Reflex Angle

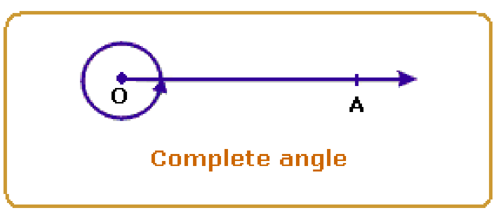
An angle whose measure is more than 180^o and less than 360^o is called a reflex angle.

It is written as ref. ∠AOB.



Complete Angle

An angle whose measure is 360^o is called a complete angle.

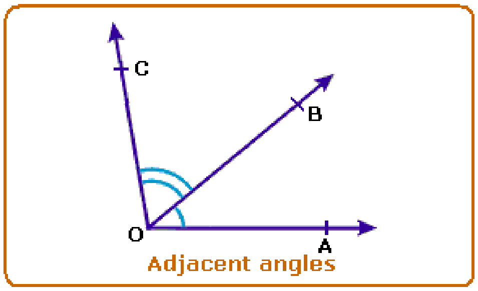


Equal Angles

Two angles are said to be equal, if they have the same measure.

Adjacent angles

Two angles having a common vertex and a common arm, such that the other arms of these angles are on opposite sides of the common arm, are called adjacent angles.



Adjacent angles

- O is the common vertex.
- $A\hat{O}B$ and $B\hat{O}C$ are adjacent angles.
- Arm BO separates the two angles.

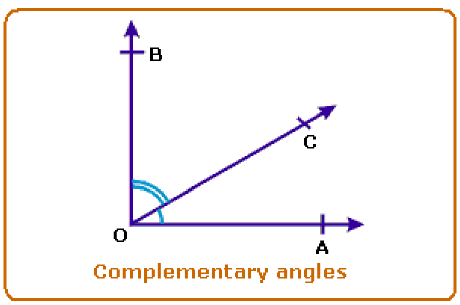
Complementary Angles

If the sum of the two angles is one right angle (i.e., 90^{o}), they are called complementary angles.

If the measure of $A\hat{O}C = a^{o}$, $C\hat{O}B = b^{o}$, then $a^{o} + b^{o} = 90^{o}$.

Therefore $A\hat{O}C$ and $C\hat{O}B$ are complementary angles.

$A\hat{O}C$ is complement of $C\hat{O}B$.



Complementary angles

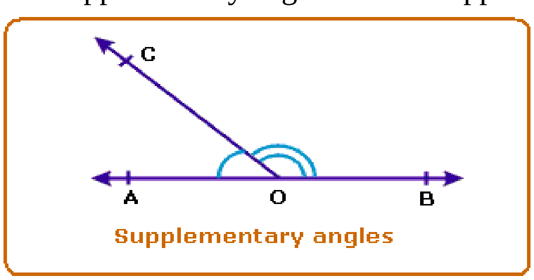
Supplementary Angles

Two angles are said to be supplementary, if the sum of their measures is 180^{o}.

Example:

Angles measuring 130^{o} and 50^{o} are supplementary angles.

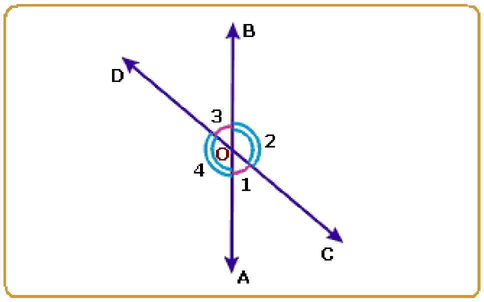
Two supplementary angles are the supplement of each other.



Supplementary angles

Vertically Opposite Angles

When two straight lines intersect each other at a point, the pairs of opposite angles so formed are called vertically opposite angles.

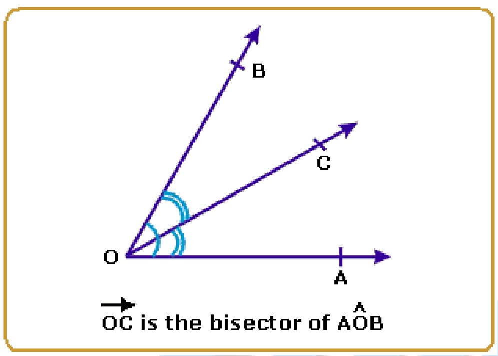


Angles $\angle 1$ and $\angle 3$ and angles $\angle 2$ and $\angle 4$ are vertically opposite angles.

Vertically opposite angles are always equal.

Bisector of an Angle

If a ray or a straight line passing through the vertex of that angle, divides the angle into two angles of equal measurement, then that line is known as the Bisector of that angle.



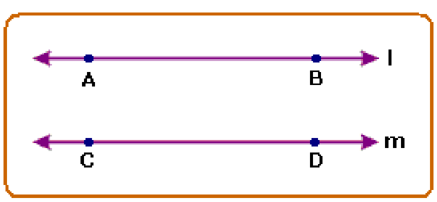
$$B\hat{O}C = C\hat{O}A$$

and $B\hat{O}C + C\hat{O}A = A\hat{O}B$

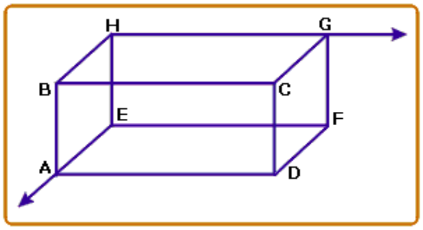
and $A\hat{O}B = 2B\hat{O}C = 2C\hat{O}A$

Parallel Lines

Two lines are parallel if they are coplanar and they do not intersect each other even if they are extended on either side.

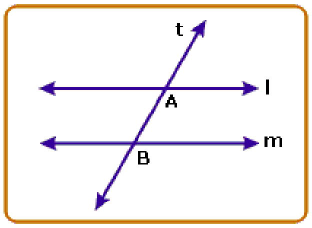


However, there are lines that do not intersect and yet are not parallel. They are skew lines. Skew lines are lines that are not coplanar and do not intersect. AE and HG are skew lines.



Transversal

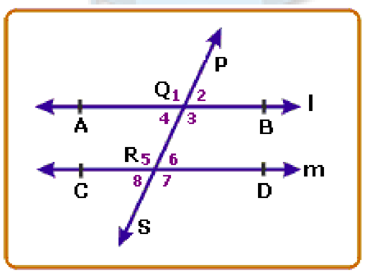
Observe the three lines 'l', 'm' and 't'.



In the diagram 'l' and 'm' are two parallel lines. 't' intersects 'l' at two distinct points 'A' and 'B' and 'm' at 'C' and 'D'. Line t is called a transversal.

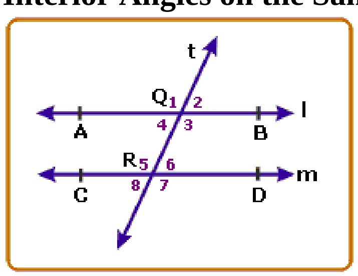
A transversal is a line that intersects (or cuts) two or more parallel lines at distinct points.

Angles Formed by a Transversal



In the diagram $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$are two parallel lines. PQRS is a transversal intersecting $\overleftrightarrow{AB}$ at Q and $\overleftrightarrow{CD}$ at R. Eight angles are formed, they are numbered from 1 to 8. By virtue of their locations, some of the angles can be paired together. The paired angles are given special names (apart from adjacent angles and vertical angles).

Interior Angles on the Same side of the Transversal



In the diagram, $A\hat{Q}R(\angle 4)$ and $Q\hat{R}C(\angle 5)$ and $B\hat{Q}R(\angle 3)$ and $Q\hat{R}D(\angle 6)$ form two pairs of interior angles on the same side of the transversal.

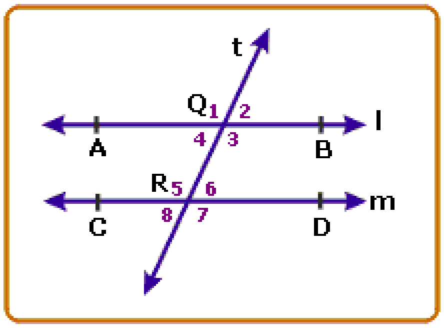
Alternate Angles

A pair of angles are said to be alternate angles if

(i) both are interior angles

(ii) they are on the opposite sides of the transversal and

(iii) they are not adjacent angles.



Alternate angles are sometimes also called alternate interior angles.

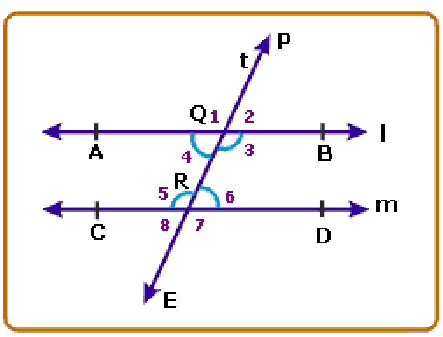
In the diagram,

- $A\hat{Q}R$ and $Q\hat{R}D$ ($\angle 4$ and $\angle 6$)
- $B\hat{Q}R$ and $Q\hat{R}C$ ($\angle 3$ and $\angle 5$)

are two pairs of alternate angles.

Corresponding Angles

- A pair of angles are said to be corresponding angles if
- One is an interior angle and the other is an exterior angle
- They are on the same side of the transversal and
- They are not adjacent angles.



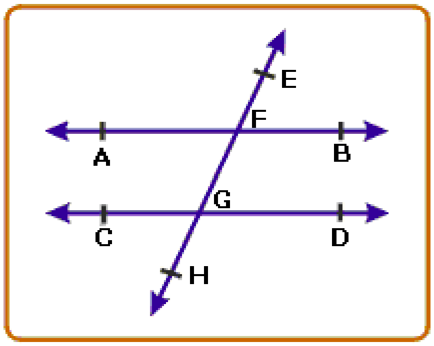
The four pairs of corresponding angles are given below.

- $A\hat{Q}P$ and $C\hat{R}Q$ ($\angle 1$ and $\angle 5$)
- $A\hat{Q}R$ and $C\hat{R}E$ ($\angle 4$ and $\angle 8$)
- $B\hat{Q}R$ and $D\hat{R}E$ ($\angle 3$ and $\angle 7$)

Parallel Lines - Theorem 1

Statement:

If a transversal intersects two parallel lines, then each pair of alternate angles are equal.



Given:

AB||CD and EFGH is a transversal.

To prove:

$A\hat{F}G = F\hat{G}D$ (one pair of interior alternate angles)

$B\hat{F}G = F\hat{G}C$ (another pair of interior alternate angles)

Proof:

$A\hat{F}G = E\hat{F}B$ (Vertically opposite angles)

But $E\hat{F}B = F\hat{G}D$ (corresponding angles)

$\therefore A\hat{F}G = F\hat{G}D$

Now $B\hat{F}G + A\hat{F}G = 180^0$ ...(i) (Linear pair)

also $F\hat{G}C + F\hat{G}D = 180^0$ ...(ii) (Linear pair)

From (i) and (ii),

$B\hat{F}G + A\hat{F}G = F\hat{G}C + F\hat{G}D$

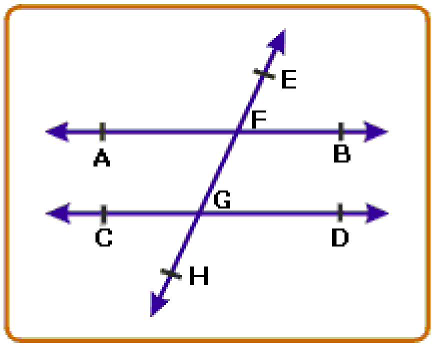
But $A\hat{F}G = F\hat{G}D$ (Proved)

$\therefore B\hat{F}G = F\hat{G}C$

Converse of Theorem 1

Statement:

If a transversal intersects two lines in such a way that a pair of alternate interior angles are equal, then the two lines are parallel.



Given:

Transversal EFGH intersects lines AB and CD such that a pair of alternate angles are equal.

$(A\hat{F}G = F\hat{G}D)$

To prove:

AB||CD

Proof:

$A\hat{F}G = F\hat{G}D$ (Given)

But $A\hat{F}G = E\hat{F}B$ (Vertically opposite angles)

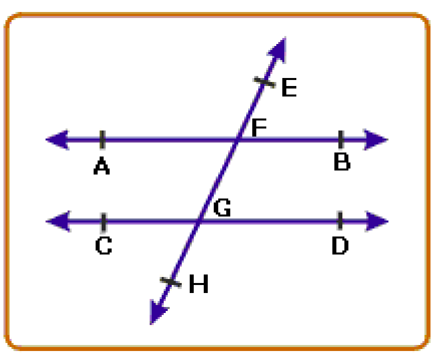
$\therefore E\hat{F}B = F\hat{G}D$ (corresponding angles)

$\therefore$ AB||CD (corresponding angles axiom)

Parallel Lines - Theorem 2

Statement:

If a transversal intersects two parallel lines, then each pair of consecutive interior angles are supplementary.



Given:
AB||CD, EFGH is a transversal.
To prove:

$B\hat{F}G + F\hat{G}D = 180^0$

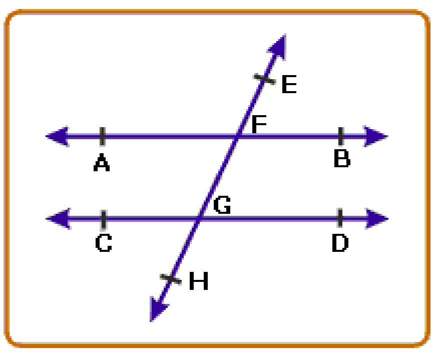
$A\hat{F}G + F\hat{G}C = 180^0$
Proof:

$E\hat{F}B + B\hat{F}G = 180^0$ (Linear pair)

$E\hat{F}B = F\hat{G}D$ (corresponding angles axiom)
$\therefore B\hat{F}G + F\hat{G}D = 180^0$ (Substitute $F\hat{G}D$ for $E\hat{F}B$)
Similarly, we can prove that

$A\hat{F}G + F\hat{G}C = 180^0$

Converse of Theorem 2
Statement:
If a transversal intersects two lines in such a way that a pair of consecutive interior angles are supplementary, then the two lines are parallel.



Given:
Transversal EFGH intersects lines AB and CD at F and G such that $B\hat{F}G$ and $F\hat{G}D$ are supplementary.

(i.e., $B\hat{F}G + F\hat{G}D = 180^0$)
To prove:
AB || CD
Proof:

$E\hat{F}B + B\hat{F}G = 180^0$ …(i) [Linear pair (ray FB stands on EFGH)]
(Corresponding angles postulate)

$F\hat{G}D + B\hat{F}G = 180^0$ …(ii) (given)

From (i) and (ii), we have

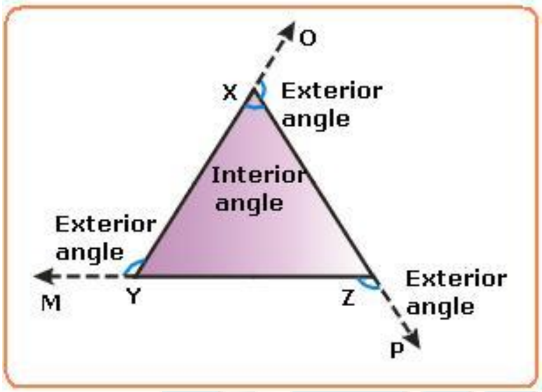
$E\hat{F}B + B\hat{F}G = F\hat{G}D + B\hat{F}G$

$\therefore E\hat{F}B = F\hat{G}D$ (Subtract $B\hat{F}G$ from both sides)

These are corresponding angles.

∴ AB || CD

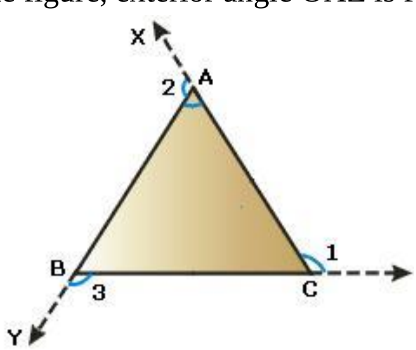
Interior and Exterior Angles of a Triangle



In a triangle when we refer to an angle, we mean the angle bounded by the two sides. The three angles lie in the interior region of the triangle. These angles are called the interior angles of the triangle. Now observe the triangle in which the sides are produced.

In the fig, $\overline{YX}$ is extended to O. An angle OXZ is formed. $\overline{XZ}$ is produced to P forming an angle YZP. Similarly, $\overline{ZY}$ is produced to M forming angle MYX. These angles OXZ, YZP and MYX are called exterior angles of Δ ABC

As the triangle has three sides, there can be three exterior angles. The interior angles opposite to the vertices where the exterior angles are formed, are called the interior opposite angles. Thus in the figure, exterior angle OXZ is formed at X. Its interior opposite angles are $X\hat{Y}Z$ *and* $X\hat{Z}Y$.



In the figure shown above,

For exterior angle 1; the interior opposite angles are $B\hat{A}C$ and $A\hat{B}C$.

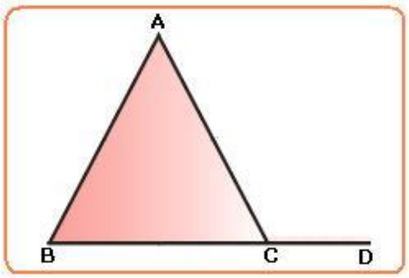
For exterior angle 2; the interior opposite angles are $A\hat{B}C$ and $A\hat{C}B$.

For exterior angle 3; the interior opposite angles are $B\hat{A}C$ and $A\hat{C}B$.

Triangles - Theorem 1

Statement:

If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the interior opposite angles.



Given:

In ΔABC, side BC is produced to D and $A\hat{C}D$ is the exterior angle formed.

$A\hat{B}C$ and $B\hat{A}C$ are the interior opposite angles.

To prove:

$A\hat{C}D = A\hat{B}C + B\hat{A}C$

Proof:

$A\hat{B}C + B\hat{A}C + A\hat{C}B = 180^0$ …(i) (Theorem)

Also $A\hat{C}B + A\hat{C}D = 180^0$ ...(ii) (Linear pair)

From (i) and (ii),

$A\hat{B}C + B\hat{A}C + A\hat{C}B = A\hat{C}B + A\hat{C}D - A\hat{C}B$

Subtract $A\hat{C}B$ from both sides,

$\therefore A\hat{B}C + B\hat{A}C = A\hat{C}D$

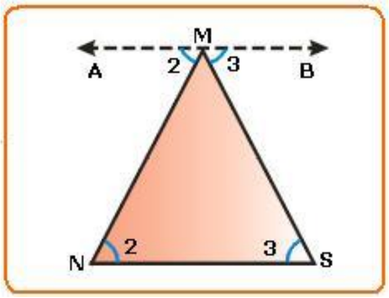
Angle Sum Property

A triangle is a plane closed geometric figure formed by three line segments joining three non-collinear points.

Triangles - Theorem 2

Statement:

The sum of the three angles of a triangle is 180°.



Given:

A triangle MNS.

To prove:

$\hat{M} + \hat{N} + \hat{S} = 180^0$

Construction:

Using a scale through the vertex M, draw a line $\overleftrightarrow{AB}$ parallel to the base $\overline{NS}$.

Proof:

Now $\overline{NS} \,||\, \overleftrightarrow{AB}$

MN is a transversal.

$\therefore A\hat{M}N = M\hat{N}S \quad ...(1)$ (Alternate angles)

Similarly $\overleftrightarrow{AB} \,||\, \overline{NS}$ and MN.

$\therefore B\hat{M}S = M\hat{S}N$. ...(2) (Alternate angles)

From the figure,

$A\hat{M}N + N\hat{M}S + B\hat{M}S = 180^0$ ($\because \overleftrightarrow{AB}$ is a straight line and sum of the angles at M = 180°)

From (1) and (2),

i.e., $M\hat{N}S + N\hat{M}S + M\hat{S}N = 180^0$ (by substituting $M\hat{N}S$ and $M\hat{S}N$)

Thus it is proved that sum of the measures of the three angles of a triangle is equal to 180° or two right angles.