

Activity 9

OBJECTIVE

To verify the algebraic identity :

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

METHOD OF CONSTRUCTION

1. Make a cube of side a units and another cube of side b units as shown in Fig. 1 and Fig. 2 by using acrylic sheet and cellotape/adhesive.
2. Make a cuboid of dimensions $a \times a \times b$ [see Fig. 3].
3. Make a cuboid of dimensions $a \times b \times b$ [see Fig. 4].
4. Arrange these cubes and cuboids as shown in Fig. 5.

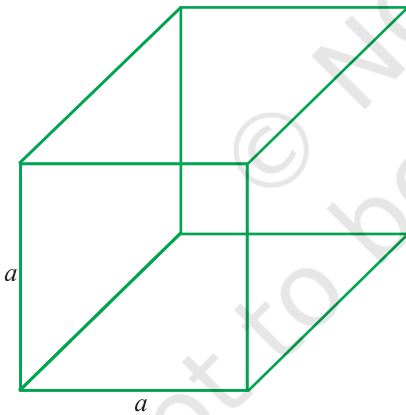


Fig. 1

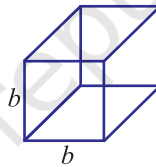


Fig. 2

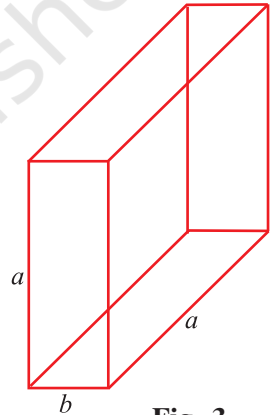


Fig. 3

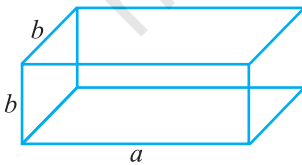


Fig. 4

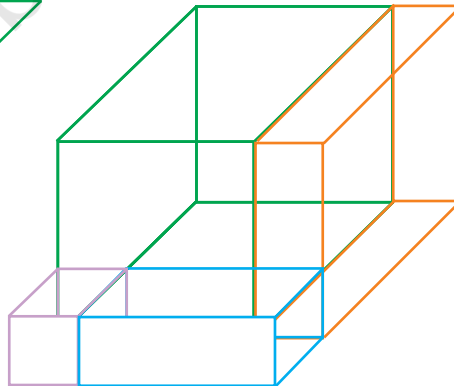


Fig. 5

DEMONSTRATION

Volume of cube in Fig. 1 = a^3

Volume of cube in Fig. 2 = b^3

Volume of cuboid in Fig. 3 = a^2b

Volume of cuboid in Fig. 4 = ab^2

Volume of solid in Fig. 5 = $a^3 + b^3 + a^2b + ab^2$
 $= (a+b)(a^2 + b^2)$

Removing cuboids of volumes a^2b and ab^2 , i.e., $ab(a+b)$ from solid obtained in Fig. 5, we get the solid in Fig. 6.

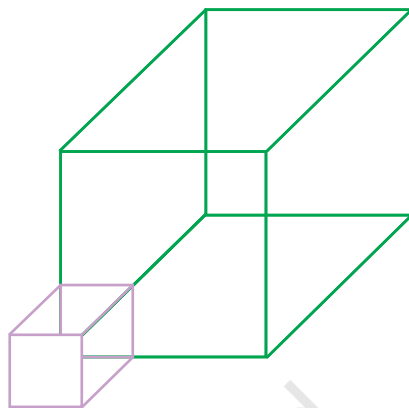


Fig. 6

Volume of solid in Fig. 6 = $a^3 + b^3$.

$$\begin{aligned}\text{Therefore, } a^3 + b^3 &= (a+b)(a^2 + b^2) - ab(a+b) \\ &= (a+b)(a^2 + b^2 - ab)\end{aligned}$$

Here, volumes are in cubic units.

OBSERVATION

On actual measurement:

$$a = \dots\dots\dots, \quad b = \dots\dots\dots,$$

$$\text{So, } a^3 = \dots\dots\dots, \quad b^3 = \dots\dots\dots, \quad (a+b) = \dots\dots\dots, \quad (a+b)a^2 = \dots\dots\dots,$$

$$(a+b)b^2 = \dots\dots\dots, \quad a^2b = \dots\dots\dots, \quad ab^2 = \dots\dots\dots,$$

$$ab(a+b) = \dots\dots\dots,$$

$$\text{Therefore, } a^3 + b^3 = (a+b)(a^2 + b^2 - ab).$$

APPLICATION

The identity may be used in simplification and factorisation of algebraic expressions.