

Activity 8

OBJECTIVE

To verify the algebraic identity

$$(a - b)^3 = a^3 - b^3 - 3(a - b)ab$$

MATERIAL REQUIRED

Acrylic sheet, coloured papers, saw, sketch pens, adhesive, Cello-tape.

METHOD OF CONSTRUCTION

1. Make a cube of side $(a - b)$ units ($a > b$) using acrylic sheet and cello-tape/ adhesive [see Fig. 1].
2. Make three cuboids each of dimensions $(a - b) \times a \times b$ and one cube of side b units using acrylic sheet and cello-tape [see Fig. 2 and Fig. 3].
3. Arrange the cubes and cuboids as shown in Fig. 4.

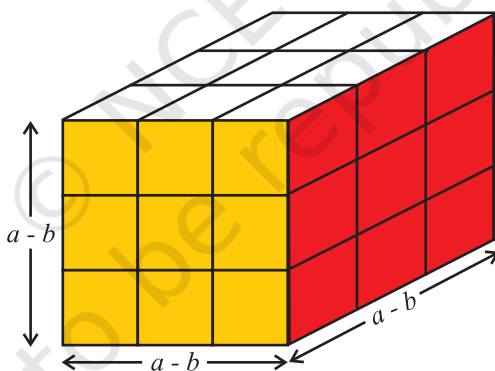


Fig. 1

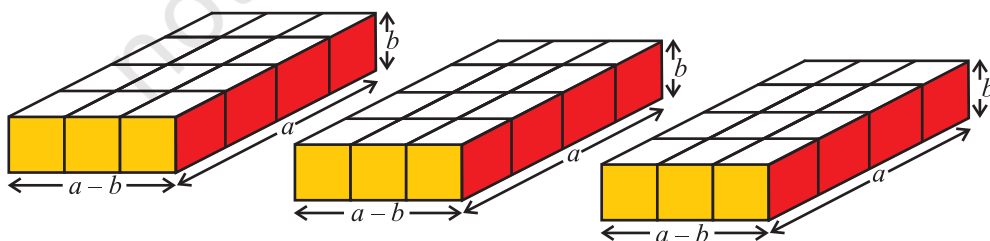


Fig. 2

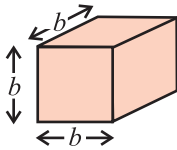


Fig. 3

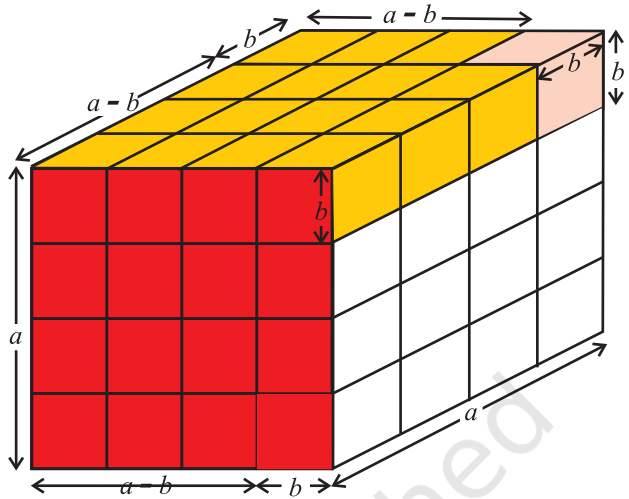


Fig. 4

DEMONSTRATION

Volume of the cube of side $(a - b)$ units in Fig. 1 = $(a - b)^3$

Volume of a cuboid in Fig. 2 = $(a - b) ab$

Volume of three cuboids in Fig. 2 = $3 (a - b) ab$

Volume of the cube of side b in Fig. 3 = b^3

Volume of the solid in Fig. 4 = $(a - b)^3 + (a - b) ab + (a - b) ab + (a - b) ab + b^3$
 $= (a - b)^3 + 3(a - b) ab + b^3$ (1)

Also, the solid obtained in Fig. 4 is a cube of side a

Therefore, its volume = a^3 (2)

From (1) and (2),

$$(a - b)^3 + 3(a - b) ab + b^3 = a^3$$

$$\text{or } (a - b)^3 = a^3 - b^3 - 3ab (a - b).$$

Here, volume is in cubic units.

OBSERVATION

On actual measurement:

$$a = \dots\dots\dots, \quad b = \dots\dots\dots, \quad a-b = \dots\dots\dots,$$

$$\text{So, } a^3 = \dots\dots\dots, \quad ab = \dots\dots\dots,$$

$$b^3 = \dots\dots\dots, \quad ab(a-b) = \dots\dots\dots,$$

$$3ab(a-b) = \dots\dots\dots, \quad (a-b)^3 = \dots\dots\dots,$$

$$\text{Therefore, } (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

NOTE

APPLICATION

The identity may be used for

1. calculating cube of a number expressed as a difference of two convenient numbers
2. simplification and factorisation of algebraic expressions.

This identity can also be expressed as :

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3.$$