1. Introduction

A rigid body generally means a hard solid object having a definite shape and size. But in reality, bodies can be stretched, compressed and bent. Even the appreciably rigid steel bar can be deformed when a sufficiently large external force is applied on it. This means that solid bodies are not perfectly rigid. A solid has definite shape and size. In order to change (or deform) the shape or size of a body, a force is required.

2. Deforming Force

A force which produces a change in configuration (size or shape) of the object on applying it, is called a deforming force.

3. Elasticity

Elasticity is that property of the object by virtue of which it regains its original configuration after the removal of the deforming force.

For example, if we stretch a rubber band and release it, it snaps back to its original length.

4. Perfectly Elastic Body

Those bodies which regain its original configuration immediately and completely after the removal of deforming force are called perfectly elastic bodies. The nearest approach to a perfectly elastic body is quartz fibre.

5. Plasticity

If a body does not regains its original size and shape completely and immediately after the removal of deforming force, it is said to be a plastic body and this property is called plasticity.

6. Perfectly plastic body

That body which does not regain its original configuration at all on the removal of deforming force are called perfectly plastic bodies.Putty and paraffin wax are nearly perfectly plastic bodies.

7. Stress

If a body gets deformed under the action of an external force, then at each section of the body an internal force of reaction is set up which tends to restore the body into its original state.

7.1 Definition

The internal restoring force set up per unit area of cross section of the deformed body is called stress.

7.2 Mathematical Form

$$\text{Stress} = \frac{\text{Applied Force}}{Area}$$

Its unit is N/m2 or Pascal.

Its dimensional formula is $[ML^{-1}T^{-2}]$.

7.3 Types of stress

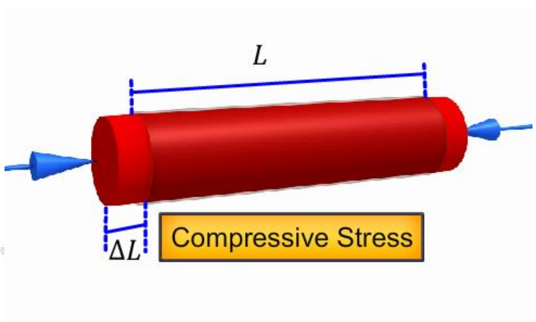
There are three different types of stress

1. Longitudinal Stress

If deforming force is applied normal to the area of cross section, then the stress is called longitudinal stress. It is further categorized in two types

(a) **Tensile stress** If there is an increase in length of the object under the effect of applied force, then stress is called tensile stress.

(b) **Compressional stress** If there is a decrease in length of the object under the effect of applied force, then stress is called compression stress.



2. Tangential or Shearing Stress

If deforming force acts tangentially to the surface of a body, it produces a change in the shape of the body. The tangential force applied per unit area is called tangential stress.

3. Normal Stress

If a body is subjected to a uniform force from all sides, then the corresponding stress is called hydrostatic stress.

8.Strain

When a deforming force acts on a body, the body undergoes a change in its shape and size. The fractional change in configuration is called strain.

8.1 Mathematical Equation

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

It has no unit and it is a dimensionless quantity.

According to the change in configuration, the strain is of three types

(1) longitudinal strain$=\frac{\text{change in length}}{\text{original length}}$

(2) Volumetric strain $=\frac{\text{change in volume}}{\text{Original volume}}$

(3) $Shearing\ strain \frac{\text{tangential applied force}}{\text{Area of face}}$

9. Hooke's Law

Robert Hook found that within the elastic limit, the stress is directly proportional to strain. Thus we have

$$\boldsymbol{stress \propto strain}$$

or

$$\boldsymbol{stress = K.strain}$$

where K is the constant of proportionality called "Elastic Modulus" of the material.

There are some materials that do not obey Hooke's law like rubber, human's muscle.

9.1 Types of Modulus of rigidity

9.1.1 Young's Modulus of rigidity (Y)

It is defined as the ratio of normal stress to the longitudinal strain within the elastic limit.

$$Y = \frac{longitudinal\ stress}{\text{Longitudinal strain}}$$

It has same units as stress because strain does not have any unit. Y is measured in N/m^2 or Pa.

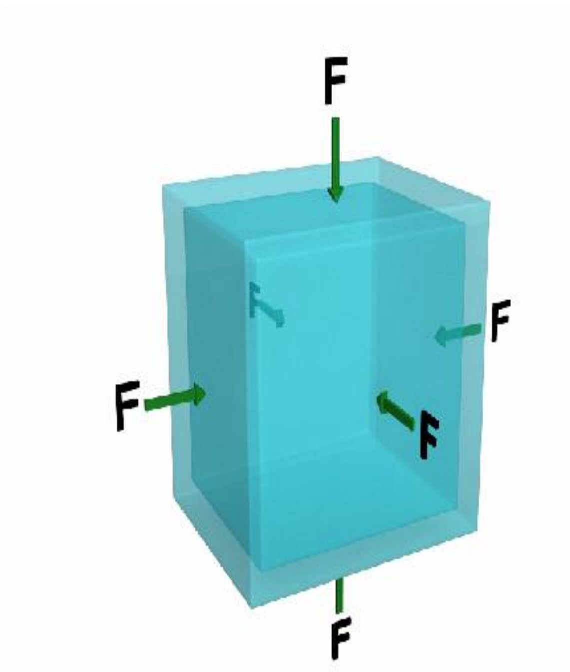
Metals generally have large values of Young's modulus compare to other materials. In scientific terms, the higher the Young's modulus of the material the more elastic it is.

9.1.2 Bulk Modulus of Rigidity

$$\kappa = \frac{normal\ stress}{\text{Volumetric strain}}$$

or $\kappa = \frac{-F/A}{\Delta V/V}$ = - p$V/\Delta V$

The SI unit of bulk modulus is N/m^2



Compressibility

Compressibility of a material is the reciprocal of its bulk modulus of elasticity. Compressibility (C) = 1 / k

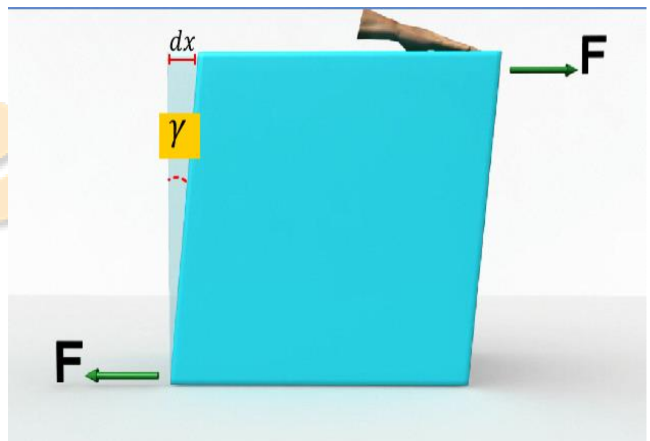
Its SI unit is N-1m 2 and CGS unit is $dyne^{-1}$ cm^2 .

9.1.3 Modulus of rigidity or shear Modulus (η)

$$\eta = \frac{tangential\ stress}{\text{shear strain}}$$

$$\eta = \frac{\frac{F}{A}}{\text{Y}} = \frac{F}{A\text{Y}}$$

$$\eta = \frac{F}{\text{AY}}$$



The SI unit of shear modulus is N/m^2

The shear modulus of a material is always considerably smaller than the Young's modulus for it.

10. Limit of elasticity

The maximum value of deforming force for which elasticity is present in the body is called its limit of elasticity.

11.Stress- strain Curve

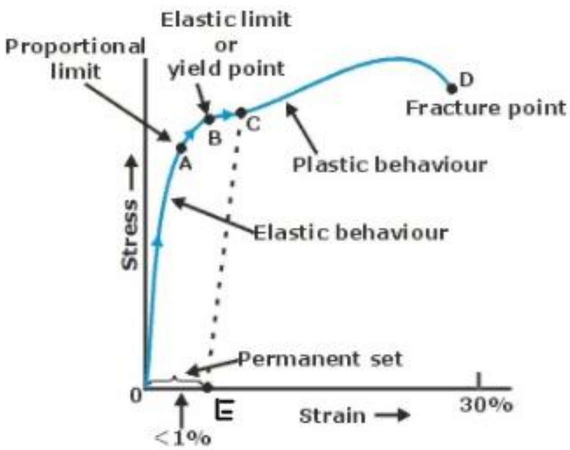


Figure shows the stress-strain curve for a metal wire which is gradually being loaded.

(a) The initial part OA of the graph is a straight line indicating that stress is proportional to strain. Upto the point A, Hooke's law is obeyed. The point A is called the proportional limit. In this region, the wire is perfectly elastic.

(b) After the point A, the stress is not proportional to strain and a curved portion AB is obtained. However, if the load is removed at any point between O and B, the curve is retraced along BAO and the wire attains its original length. The portion OB of the graph is called elastic region and the point B is called elastic limit or yield point. The stress corresponding to B is called yield strength.

(c) Beyond the point B, the strain increases more rapidly than stress. If the load is removed at any point C, the wire does not come back to its original length but traces dashed line. Even on reducing the stress to zero, a residual strain equal to OE is left in the wire. The material is said to have acquired a permanent set. The fact that stress-strain curve is not retraced on reversing the strain is called elastic hysteresis.

(d) If the load is increased beyond the point C, there is large increase in the strain or the length of the wire. In this region, the constrictions (called necks and waists) develop at few points along the length of the wire and the wire breaks ultimately at the point D, called the fracture point.

In the region between B and D, the length of the wire goes on increasing even without any addition of load. This region is called plastic region and material is said to undergo plastic flow or plastic deformation. The stress corresponding to the braking point is called ultimate strength or tensile strength of the material.

12. Elastic after Effect

The bodies return to their original state on the removal of the deforming force. Some bodies return to their original state immediately after the removal of the deforming force while some bodies take longer time to do so. The delay in regaining the original state by a body on the removal of the deforming force is called elastic after effect.

13. Elastic Fatigue

The property of an elastic body by virtue of which its behavior becomes less elastic under the action of repeated alternating deforming force is called elastic fatigue.

14. Ductile Materials

The materials which have large plastic range of extension are called ductile materials. Such materials undergo an irreversible increase in length before snapping. So they can be drawn into thin wires. For e.g. copper, silver, iron, aluminium etc.

15 Brittle Materials

The materials which have very small range of plastic extension are called brittle materials. Such materials break as soon as the stress is increased beyond the elastic limit. For e.g. cast iron, glass, ceramics etc.

16. Elastomers

The materials for which strain produced is much larger than the stress applied, with in the limit of elasticity are called elastomers, e.g., rubber, the elastic tissue of aorta, the large vessel carrying blood from heart. etc. Elastomers have no plastic range.

17. Elastic Potential Energy of stretched wire

When a wire is stretched, interatomic forces come into play which opposes the change. Work has to done against these restoring forces. The work done in stretching the wire is stored in it as its elastic potential energy.

18. Poisson's Ratio

When a deforming force is applied at the free end of a suspended wire of length 1 and diameter D, then its length increases by Δl but its diameter decreases by ΔD. Now two types of strains are produced by a single force.

(i) Longitudinal strain = $\frac{\Delta l}{l}$

(ii) Lateral strain = $\frac{-\Delta D}{D}$

∴ Poisson's Ratio (σ) = $\frac{Lateral\ strain}{longitudinal\ strain}$=

$$= \frac{\frac{-\Delta D}{D}}{\frac{\Delta l}{l}} = -\frac{l\Delta D}{D\Delta l}$$

The negative sign shows that longitudinal and lateral strains are in opposite sense.

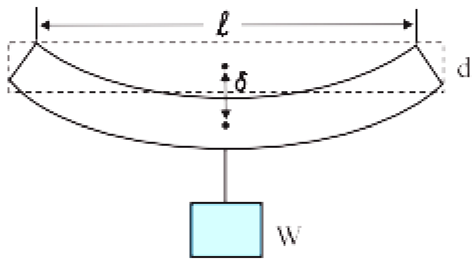
As Poisson's ratio is the ratio of two strains, it has no units and dimensions.

The theoretical value of Poisson's ratio lies between – 1 and 0.5. Its practical value lies between 0 and 0.5

19. Applications of elasticity

The elastic behavior of materials plays an important role in everyday life. All engineering designs require precise knowledge of the elastic behavior of materials. For example while designing a building, the structural design of the columns, beams and supports require knowledge of strength of materials used.

A bridge has to be designed such that it can withstand the load of the flowing traffic, the force of winds and its own weight. Similarly, in the design of buildings use of beams and columns is very common. In both the cases, the overcoming of the problem of bending of beam under a load is of prime importance. The beam should not bend too much or break. Let us consider the case of a beam loaded at the centre and supported near its ends as shown in Fig.



A bar of length l, breadth b, and depth d when loaded at the centre by a load W sags by an amount given by $\delta = W l^3/(4bd^3Y)$

Bending can be reduced by using a material with a large Young's modulus Y. Depression can be decreased more effectively by increasing the depth d rather than the breadth b. But a deep bar has a tendency to bend under the weight of a moving traffic, hence a better choice is to have a bar of I-shaped cross section. This section provides a large load bearing surface and enough depth to prevent bending. Also this shape reduces the weight of the beam without sacrificing its strength and hence reduces the cost.