WORK, POWER & ENERGY

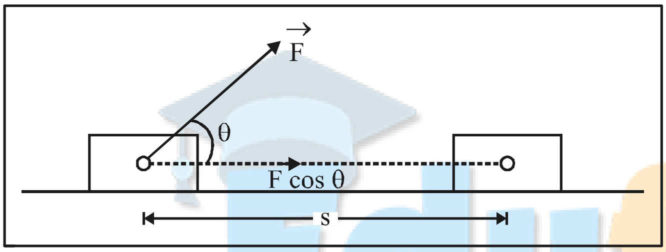
1. WORK

In Physics, work stands for 'mechanical work'.

Work is said to be done by a force when the body is displaced actually through some distance in the direction of the applied force.

However, when there is no displacement in the direction of the applied force, no work is said to be done, i.e., work done is zero, when displacement of the body in the direction of the force is zero.

Suppose a constant force $\vec{F}$ acting on a body produces a displacement $\vec{s}$ in the body along the positive x-direction, figure



If θ is the angle which $\vec{F}$ makes with the positive x–direction of the displacement, then the component of $\vec{F}$ in the direction of displacement is (F cos θ). As work done by the force is the product of component of force in the direction of the displcement and the magnitude of the displacement,

$$\therefore \quad \boxed{W = (F\cos\theta)\, s} \qquad ...(1)$$

If displacement is in the direction of force applied, θ = 0°. From (1), W = (F cos 0°) s = F s

Equation (1) can be rewritten as $\boxed{W = \vec{F}.\vec{s}}$ ...(2)

Thus, work done by a force is the dot product of force and displacement.

In terms of rectangular cmponent, $\vec{F}$ and $\vec{s}$, may written as

$$\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z \text{ and } \vec{s} = \hat{i}x + \hat{j}y + \hat{k}z$$

From (2), $W = \vec{F}.\vec{s}$

$$W = \left(\hat{i}F_x + \hat{j}F_y + \hat{k}F_z\right).\left(\hat{i}x + \hat{j}y + \hat{k}z\right)$$

$$\boxed{W = xF_x + yF_y + zF_z}$$

Obviously, work is a scalar quantity, i.e., it has magnitude only and no direction. However, work done by a force can be positive or negative or zero.

2. DIMENSIONS AND UNITS OF WORK

As work = force × distance $\therefore$ $W = (M^1 L^1 T^{-2}) \times L$

$$\boxed{W = \left[M^1 L^2 T^{-2}\right]}$$

This is the dimensional formula of work.

The **units** of work are of two types :

1. Absolute units 2. Gravitational units

(a) Absolute unit

1. Joule. It is the absolute unit of work on SI.

Work done is said to be one joule, when a force of one newton actually moves a body through a distance of one metre in the direction of applied force.

From W = F cos θ

1 joule = 1 newton × 1 metre × cos 0° = 1 N–m

2. Erg. It is the absolute unit of work on cgs system.

Work done is said to be one erg, when a force of one dyne actually moves a body through a distance of one cm.in the direction of applied force.

From W = Fs cos θ

1 eg = 1 dyne × 1 cm × cos 0°

(b) Gravitational units

These are also called the practical units of work.

1. Kilogram-metre (kg–m). It is the gravitational unit of work on SI.

Work done is said to be one kg–m, when a force of 1 kg f move a body through a distance of 1 m in the direction of the applied force.

From W = F s cos θ

1 kg–m = 1 kg f × 1 m × cos 0° = 9.8 N × 1 m = 9.8 joule, i.e.,

$$\boxed{1\,kg - m = 9.8\,J}$$

2. Gram-centimetre (g-cm). It is the gravitational unit of work on cgs system.

Work done is said to be one g-cm, when a force of 1 g f moves a body through a distance of 1 cm. in the direction of the applied force.

From W = F s cos θ

1 g-cm = 1 g f × 1 cm × cos 0°

1 g-cm = 980 dyne × 1 cm × 1

$\boxed{1\,g - m = 980\,ergs}$

3. NATURE OF WORK DONE

Although work done is a scalar quantity, its value may be positive, negative, negative or even zero, as detailed below:

(a) Positive work

As W = $\vec{F}.\vec{s}$ = F s cos θ

∴ when θ is acute (< 90°), cos θ is positive. Hence, work done is positive.

For example :

(i) When a body falls freely under the action of gravity, θ = 0°, cos θ = cos 0° = + 1. Therefore, work done by gravity on a body falling freely is positive.

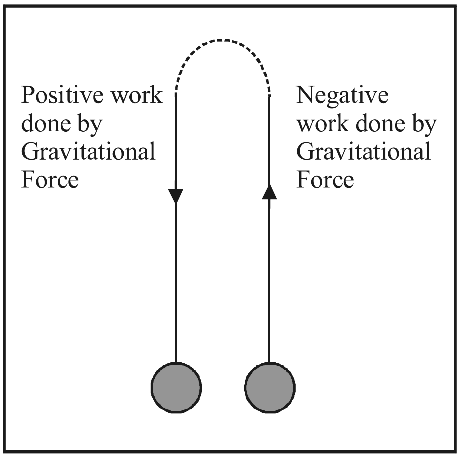
(b) Negative work

As W = $\vec{F}.\vec{s}$ = F s cos θ

∴ When θ is obtuse (> 90°), cos θ is negative. Hence, work done is negative.

For example :

(i) When a body is thrown up, its motion is opposed by gravity. The angle θ between gravitational force $\vec{F}$ and the displacement $\vec{s}$ is 180°. As cos θ = cos 180° = –1, therefore, work done by gravity on a body moving upwards is negative.



(c) Zero work

When force applied $\vec{F}$ or the displacement $\vec{s}$ or both are zero, work done W = F s cos θ is zero. Again, when angle θ between $\vec{F}$ and $\vec{s}$ is 90°, cos θ = cos 90° = 0. Therefore work done is zero.

For example :

When we push hard against a wall, the force we exert on the wall does no work, because $\vec{s} = 0$. However, in this process, our muscles are contracting and relaxing alternately and internal energy is being used up. That is why we do get tired.

4. WORK DONE BY A VARIABLE FORCE

(a) Graphical Method

A constant force is rare. It is the variable force which is encountered more commonly. We can, therefore, learn to calculate work done by a variable force, let us consider a force acting along the fixed direction, say x–axis, but having a variable magnitude.

We have to calculate work done in moving the body from A to B under the action of this variable force. To do this, we assume that the entire displacement from A to B is made up of a large number of infinitesimal displacements. One such displacement shown in figure from P to Q.

As the displacement PQ = dx is infinitesimally small, we consider that all along this displacement, force is constant in magnitude (= PS) as well in same direction.

∴ Small amount of work done in moving the body from P to Q is

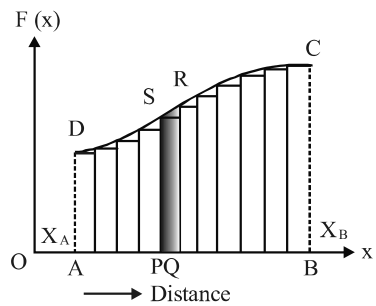
dW = F × dx = (PS) (PQ) = area of strip PQRS

Total work done in moving the body from A to B is given by

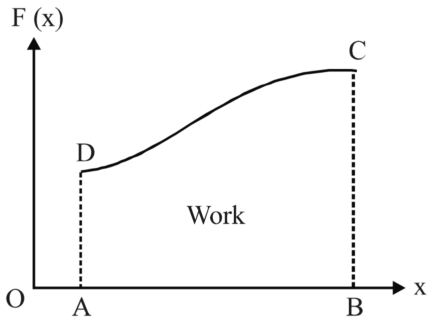
W = Σ dW

W = Σ F × dx

If the displacement are allowed to approach zero, then the number of terms in the sum increases without limit. And the sum approaches a definite value equal to the area under the curve CD.



(a)



(b)

Hence, we may rewrite, $W = \lim_{dx \to x} \Sigma F(dx)$

In the language of integral calculus, we may write it as

$$W = \int_{x_A}^{x_B} F(dx), \text{ where } x_A = O_A \text{ and } x_B = OB$$

$$W = \int_{x_A}^{x_B} \text{area of the strip PQRS}$$

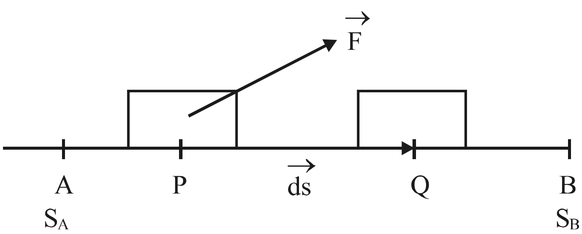
= total area under the curve between F and x-axis from $x = x_A$ to $x = x_B$

$$\boxed{W = \text{Area ABCDA}}$$

Hence, work done by a variable force is numerically equal to the area under the force curve and the displacement axis.

Mathematical Treatment (of work done by a variable force).

Suppose we have to calculate work done in moving a body from a point A (S_A) to point B (S_B) under the action of a varying force, figure. Here, S_A and S_B are the distance of the points A and B with respect to some reference point.



At any stage, suppose the body is at P, where force on the body is $\vec{F}$. Under the action of this force, let the body undergo an infinitesimally small displacement $\overrightarrow{PQ} = \overrightarrow{ds}$. During such a small displacement, if we assume that the force remains constant, then small amount of work done in moving the body from P to Q is

$$dW = \vec{F}.\overrightarrow{ds}$$

When $\overrightarrow{ds} \to 0$, total work done in moving the body from A to B can be obtained by integrating the above expression between S_A and S_B.

$$\therefore \quad \boxed{W = \int_{S_A}^{S_B} \vec{F}.\overrightarrow{ds}}$$

5. CONSERVATIVE & NON-CONSERVATIVE FORCES

Conservative force

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body, and not on the nature of path followed between the initial and the final positions.

This means, work done by or against a conservative force in moving a body over any path between fixed initial and final positions will be the same.

For example, gravitational force is a conservative force.

Properties of Conservative forces :

1. Work done by or against a conservative force, in moving a body from one position to the other depends only on the initial position and final position of the body.
2. Work done by or against a conservative force does not depend upon the nature of the path followed by the body in going from initial position to the final position.
3. Work done by or against a conservative force in moving a body through any round trip (i.e., closed path, where final position coincides with the initial position of the body) is always zero.

Non-conservative Forces

A force is said to be non-conservative, if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions.

For example, frictional forces are non-conservative forces.

6. POWER

Power of a person or machine is defined as the time rate at which work is done by it.

$$\text{i.e., Power} = \text{Rate of doing work} = \frac{\text{work done}}{\text{time taken}}$$

Thus, power of a body measures how fast it can do the work.

$$P = \frac{dW}{dt}$$

Now, $dW = \vec{F}.\overrightarrow{ds}$, where $\vec{F}$ is the force applied and $\overrightarrow{ds}$ is the small displacement.

$$\therefore \quad P = \frac{\vec{F}.\overrightarrow{ds}}{dt}$$

But $\frac{\overrightarrow{ds}}{dt} = \vec{v}$, the instantaneous velocity.

$$\therefore \quad \boxed{P = \vec{F}.\vec{v}}$$

Dimensions of power can be deduced as :

$$\boxed{P = \frac{W}{t} = \frac{M^1L^2T^{-}}{T^1} = \left[M^1L^2T^{-3}\right]}$$

Units of power

The absolute unit of power in SI is **watt**, which is denoted by W.

From P = W/t

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{sec}}, \text{ i.e., } \boxed{1W = 1Js^{-1}}$$

Power of a body is said to be one watt, if it can do one joule of work in one second.

$$\boxed{1 \text{ h.p.} = 746 \text{ W}}$$

7. ENERGY

Energy of a body is defined as the capacity or ability of the body to do the work.

8. KINETIC ENERGY

The kinetic energy of a boyd is the energy possessed by the body by virtue of its motion.

For example :

(i) A bullet fired from a gun can pierce through a target on account of kinetic energy of the bullet.

(ii) Wind mills work on the kinetic energy of air. For example, sailing ships use the kinetic energy of wind.

(iii) Water mills work on the kinetic energy of water. For example, fast flowing stream has been used to grind corn.

(iv) A nail is driven into a wooden block on account of kinetic energy of the hammer striking the nail.

Formula for Kinetic Energy

Kinetic Energy of a body can be obtained either from

(i) the amount of work done in stopping the moving body, or from

(ii) the amount of work done in giving the present velocity today he body from the state of rest.

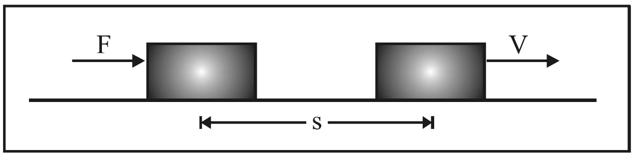
Let us use the second method :

suppose m = mass of a body at rest (i.e., u = 0).

F = Force applied on the body

a = acceleration produced in the body in the direction of force applied.

v = velocity acquired by the body in moving through a distance s, figure



From v – u = 2 a s

$v^2 - 0 = 2$ as

$$a = \frac{v}{2s}$$

As F = m a $\therefore$ using, $F = m\left(\frac{v^2}{2s}\right)$

Work done on the body, W = Force × distance

$$W = m\frac{v}{2s} \times s$$

$$W = \frac{1}{} m v$$

This work done on the body is a measure of kinetic energy (K.E.) acquired by the body,

$$\therefore \quad \boxed{\text{K.E. of body} = W = \frac{1}{} m v}$$

Alternative method

The formula for kinetic energy of a body is also obtained by the method of calculus :

Let m = mass of a body, which is initially at rest

(i.e., u = 0)

$\vec{F}$ = Force applied on the body,

$\overrightarrow{ds}$ = small displacement produced in the body in the direction of the force applied.

$\therefore$ Small amount of work done by the force,

$dW = \vec{F}.\overrightarrow{ds} = F\,ds\cos = 0° = F\,ds$

If a is acceleration produced by the force, then from

$F = m\,a = m\,\frac{dv}{dt}$

From, $dW = \left(m\frac{dv}{dt}\right)ds = m\left(\frac{ds}{dt}\right)dv$

$dW = m\,v\,d\,v \qquad \left(\because \frac{ds}{dt} = v\right)$

$\therefore$ Total work done by the force in increasing the velocity of the body from zero to v is

$W = \int_0^v m\,v\,d\,v = m\int_0^v v\,d\,v = m\left[\frac{2}{2}\right]_0$

$W = \frac{1}{\ }m\,v$

Thus, kinetic energy of a body is half the product of mass of the body and square of velocity of the body.

9. RELATION BETWEEN KINETIC ENERGY AND LINEAR MOMENTUM

Let m = mass of a body, v = velocity of the body.

$\therefore$ Linear momentum of the body, p = mv

and K.E. of the body $= \frac{1}{\ }mv \quad = \frac{1}{2m}(m^2v\ )$

$\therefore$ $\boxed{K.E. = \frac{p}{2m}}$

This is an important relation. It shows that a body cannot have K.E. without having linear momentum. The reverse is also true.

Further, if p = constant, K.E. $\propto \frac{1}{m}$

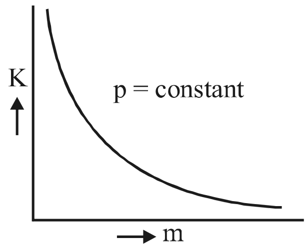
This is showin in figure (a)

If K.E. = constant, $p^2 \propto m$ or $p \propto \sqrt{m}$

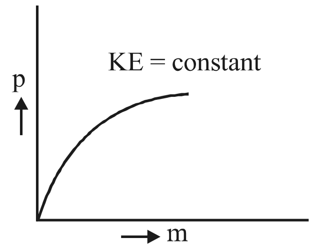
This is shown in figure (b).

If m = constant, $p^2 \propto$ K.E. or $p \propto \sqrt{K.E.}$

This is shown in figure (c)



(a)



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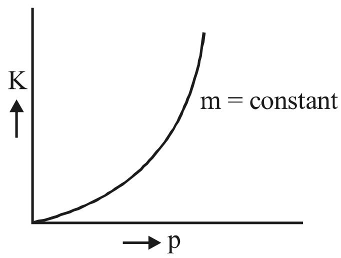


(c)

10. WORK ENERGY THEOREM OR WORK ENERGY PRINCIPLE

According to this principle, work done by net force in displacing a body is equal to change in kinetic energy of the body.

Thus, when a force does some work on a body, the kinetic energy of the body increases by the same amount. Conversely, when an opposing (retarding) force is applied on a body, its kinetic energy decreases. The decrease in kinetic energy of the body is equal to the work done by the body against the retarding force. Thus, according to work energy principle, work and kinetic energy are equivalent quantities.

Proof : To prove the work-energy theorem, we confine ourselves to motion in one dimension.

Suppose m = mass of a body, u = initial velocity of the body, F = force applied on the body along it direction of motion, a = acceleration produced in the body, v = final velocity of the body after t second.

Small amount of work done by the applied force on the body, dW = F (ds) when ds is the small distance moved by the body in the direction of the force applied.

Now, $F = ma = m\left(\frac{dv}{dt}\right)$

$\therefore \quad dW = F\,(ds) = m\left(\frac{dv}{dt}\right) ds = m\left(\frac{ds}{dt}\right) dv = mv\,dv$

$\left(\because \frac{ds}{dt} = v\right)$

Total work done by the applied force on the body in increasing its velocity from u to v is

$$W = \int_u^v m\,v\,d\,v = m\int_u^v v\,d\,v = m\left[\frac{v^2}{2}\right]_u^v$$

$$W = \frac{1}{2}m\left(v^2 - u^2\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

But $\frac{1}{2}mv^2 = K_f$ = final K.E. of the body and $\frac{1}{2}mu^2 = K_i$ = initial K.E. of the body

$\therefore \quad W = K_f - K_i$ = Increases in K.E. of body

i.e., work done on the body = increase in K.E. of body.

11. POTENTIAL ENERGY

The potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration in some field.

Thus, potential energy is the energy that can be associated with the configuration (or arrangement) of a system of objects that exert forces on one another. Obviously, if configuration of the system changes, then its potential energy changes.

Two important types of potential energy are :

1. Gravitational potential energy
2. Elastic potential energy.

11.1 Gravitational Potential Energy

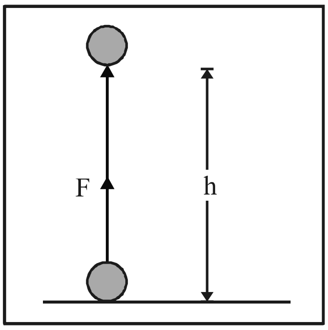
Gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

To calculate gravitational potential energy, suppose

m = mass of a body

g = acceleration due to gravity on the surface of earth.

h = height through which the body is raised, figure.



If we assume that height h is not too large and the value of g is practically constant over this height, then the force applied just to overcome gravitational attraction is

$F = mg$

As the distance moved is in the direction of the force applied, therefore,

Work done = force × distance

$W = F \times h = mgh$

Note that we have taken the upward direction to be positive. Therefore, work done by applied force = + mgh. However, work done by gravitational force = – mgh.

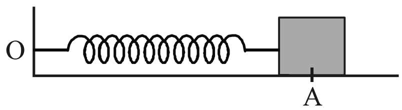
This work gets stored as potential energy. The gravitational potential energy of a body, as a function of height (h) is denoted by V (h), and it is negative of work done by the gravitational force in raising the body to that height.

Gravitational P.E. = V (h) = mgh

11.2 Potential Energy of a spring

Potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

To calculate it, consider an elastic spring OA of negligible mass. The end O of the spring is fixed to a rigid support and a body of mass m is attached to the free end A. Let the spring be oriented along x–axis and the body of mass m lie on a perfectly frictionless horizontal table.



The position of the body A, when spring is unstretched is chosen as the origin.

When the spring is compressed or elongated, it tends to recover to its original length, on account of elasticity. The force trying to bring the spring back to its original configuration is called restoring force or spring force.

For a small stretch or compression, spring obeys Hook's law, i.e., for a spring, Restoring Force ∝ stretch or compression

$-F \propto x$ or $-F = kx$

where k is a constant of the spring and is called spring constant.

It is established that for a spring, $k \propto \frac{1}{\ell}$

i.e., smaller the length of the spring, greater will be the force constant and vice-versa.

The negative sign in equation indicates that the restoring force is directed always towards the equilibrium position.

Let the body be displaced further through an infinitesimally small distance dx, against the resotring force.

∴ Small amount of work done in increasing the length of the spring by dx is

$dW = -F\,dx = kx\,dx$

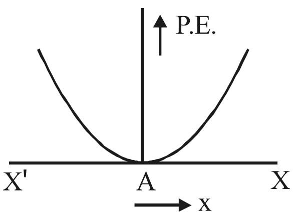
Total work done in giving displacement x to the body can be obtained by integrating from x = 0 to x = x, i.e.,

$$W = \int_{x=0}^{x=x} k\,x\,dx = k\left[\frac{x}{\ }\right]_{x=0}^{x=x} = k\left[\frac{x}{2} - 0\right] = \frac{1}{\ }kx$$

This work done is stored in the spring at the point B.

∴ $\boxed{\text{P.E. at B} = W = \frac{1}{\ }kx}$

The variation of potential energy with distance x is shown in figure



12. MECHANICAL ENERGY AND ITS CONSERVATION

The mechanical energy (E) of a body is the sum of kinetic energy (K) and potential energy (V) of the body

i.e., $\boxed{E = K + V}$

Obviously, mechanical energy of a body is a scalar quantity measured in joule.

We can show that the total mechanical energy of a system is conserved if the force, doing work on the system are conservative.

This is called the principle of conservation of total mechanical energy.

For simplicity, we assume the motion to be one dimensional only. Suppose a body undergoes a small displacement Δx under the action of a conservative force F. According to work energy theorem,

change in K.E. = work done

$\Delta K = F(x)\,\Delta x$

As the force is conservative, the potential energy function V (x) is defined as

$-\Delta V = F(x)\,\Delta x$ or $\Delta V = -F(x)\,\Delta x$

Adding, we get $\Delta K + \Delta V = 0$ or $\Delta(K + V) = 0$,

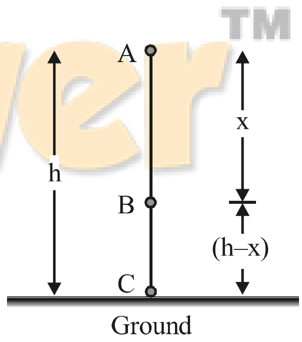
which means

$\boxed{(K + V) = E = \text{constant}}$

12.1 Illustration of the law of conservation of mechanical energy

To illustrate the law further, let us calculate kinetic energy K.E., potential nergy P.E. and total energy T.E. of a body falling freely under gravity.

Let m be the mass of the body held at A, at a height h above the ground, figure.



As the body is at rest at A, therefore,

At A : K. E. of the body = 0

P.E. of the body = mg, where g is acceleration due to gravity at A.

T.E. of the body = K.E. + P.E. = 0 + mgh

$E_1 = mgh$ ...(1)

Let the body be allowed to fall freely under under gravity, when it strikes the ground at C with a velocity v.

From $v^2 - u^2 = 2as$

$v^2 - 0 = 2(g)h$

$v^2 = 2gh$ ...(2)

∴ At C : K.E. of the body $= \frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$

P.E. of the body = mgh = mg (0) = 0

Total energy of the body = I.E. + P.E.

$E_2 = mgh + 0 = mgh$ ...(3)

In free fall, let the body cross any point B with a velocity v_1, where AB = x

From $v^2 - u^2 = 2as$

$v_1^2 - 0 = 2(g)x$ ...(4)

$v_1^2 = 2gx$

At B : K.E. of the body $= \frac{1}{2}mv_1^2 = \frac{1}{2}m(2gx) = mgx$

Height of the body at B above the ground = CB = (h – x)

∴ P.E. of the body at B = mg (h – x)

Total energy of the body at B = K.E. + P.E.

$E_3 = mgx + mg(h - x) = mgx + mgh - mgx$

$E_3 = mgh$ ...(5)

From (1), (3), (5) we find that

$$\boxed{E_1 = E_2 = E_3 = mgh}$$

13. DIFFERENT FORMS OF ENERGY

We have studies some details of potential energy and kinetic energy. These are not the ony two forms of energy. Energy may manifest itself in several other forms. Some of the examples are :

1. Heat Energy

It is the energy possessed by a body by virtue of random motion of the molecules of the body.

Heat is also associated with the force of friction. When a block of mass m sliding on a rough horizontal surface with speed v, stops over a distance x, work done by the force of kinetic friction f over a distance x is –f (x). By the work energy theorem, $\frac{1}{2}mv^2 = f(x)$. We often say that K.E. of the block is lost due to frictional force, However, when we examine the block and the horizontal surface carefully, we detect a slight increase in their temperatures. Thus, work done by friction is not lost, but it is transferred as heat energy of the system.

2. Internal Energy

It is the total energy possessed by the body by virtue of particular configuration of its molecules and also their random motion. Thus, internal energy of a body is the sum of potential energy and kinetic energy of the molecules of the body.

3. Electrical Energy

The flow of electric current causes bulbs to glow, fans to rotate and bells to ring. A definite amount of work has to be done in moving the free charge carriers in a particular direction through all the electrical appliances.

4. Chemical Energy

Chemical energy arises from the fact that the molecules participating in the chemical reaction have different binding energies. A chemical reaction is basically a rearrangement of atoms. For example, coal consists of carbon and a kilogram of it. When burnt releases 3×10^7 J of energy.

5. Nuclear Energy

It is the energy obtainable from an atomic nucleus. Two distinct modes of obtaining nuclear energy are (i) Nuclear fission (ii) Nuclear fusion.

Nuclear fission involves splitting of a heavy nucleus into two or more lighter nuclei, whereas nuclear fusion involves fusing of two or more lighter nuclei to form a heavy nucleus.

14. MASS ENERGY EQUIVALENCE

Einstein made an incredible discovery that energy can be transformed into mass and mass can be transformed into energy. One can be obtained at the cost of the other. The mass energy equivalence relation as put forth by Einstein is

$$\boxed{E = mc^2}$$

where m = mass that disappears, E = energy that appears, c = velocity of light in vacuum.

Mass and energy are not conserved separately, but are conserved as a single entity called 'mass-energy'.

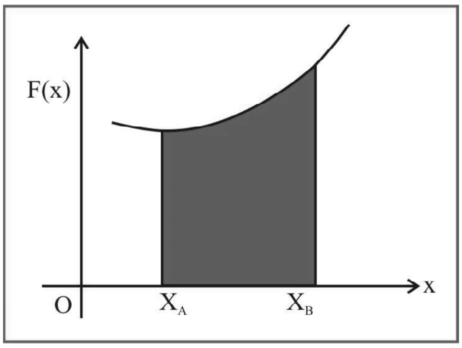
15. THE PRINCIPLE OF CONSERVATION OF ENERGY

If we account for all forms of energy, the total energy of an isolated system does not change.

The principle of conservation of energy cannot be proved as such. However, no violation of this principle has ever been observed.

16. WORK DONE BY A VARIABLE FORCE

When the force is an arbitrary function of position, we need the techniques of calculus to evaluate the work done by it. The figure shows F (x) as some function of the position x. To calculate work done by F from A to B, we find area under the graph from X_A to X_B.



Thus, the work done by a force F (x) form an initial point A to final point B is

$$W_{A\to B} = \int_{x_A}^{x_B} F_x dx$$

17. CONSERVATIVE & NON-CONSERVATIVE FORCES

17.1 Conservative Forces

There are two ways in which we can characterize a **Conservative Force :**

A force is conservative if :

- The net work done against the force in moving a mass between two points depends only on the location of two points and not on the path followed

17.2 Non-Conservative Forces

Those forces which do not satisfy the above mentioned criteria. Friction and viscous forces are the most common examples of non-conservative forces.

17.3 Conservative Forces and Potential Energy

For every conservative force, there is a corresponding potential energy function. In each cases, the potential energy expression depends only on position.

For every conservative force F_x that depends only on the position x, there is an associated potential energy function U(x). When conservative force does positive work, the potential energy of the system decreases. Work done by, conservative force is

$$F(x)\,\Delta x = -\Delta U$$

$$\Rightarrow \quad F(x) = -\Delta U / \Delta x$$

which, in the limit, becomes $F(x) = -\dfrac{dU}{dx}$

Integrating both sides for a displacement from x = a to x = b, we have :

$$U_b - U_a = -\int_a^b F(x)\,dx$$

18. DYNAMICS OF CIRCULAR MOTION

18.1 Force on the Particle

In uniform circular motion, acceleration is of magnitude v^2/r and is directed towards centre. Hence a force of magnitude mv^2/r and directed towards centre is required to keep a particle in circular motion. This force (acting towards centre) is known as the centripetal force. Centripetal force is not an extra force on a body. Whatever force is responsible for circular motion becomes the centripetal force.

Example : When a statellite revolves around the earth, the gravitational attraction of earth becomes the centripetal force for the circular motion of the satellite; when an electron revolves around the nucleus in an atom, the electrostatic attraction of nucleus becomes the centripetal force for the electron's circular motion; in case of a conical pendulum, T sin θ (component of tension) becomes the centripetal force.

18.2 Main steps for analysing forces in uniform circular motion

Take one axis along the radius of circle (i.e., in direction of acceleration) and other axis perpendicular to the radius. Resolve all the forces into components.

Net force along perpendicular axis $= 0$

Net force along radial axis (towards centre)

$$= \frac{mv^2}{r} = m\omega^2 r$$

18.3 Main steps for analysing forces in Non–uniform Circular Motion

After resolving all the forces along tangential and radial axes :

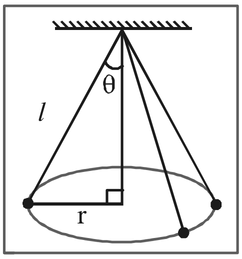
net tangential force $= F_t = m\,a_t$

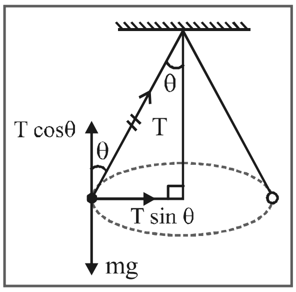
net radial force $= F_r = m\,a_r = mv^2/r$

Example of non-uniform circular motion : the motion of particle in verticle circle. If a particle is revolved in a vertical circle with the help of a string, the forces are : tension (T) towards centre and weight (mg). In case of a particle moving along the outside surface of a circular track (or sphere), the forces are : normal reaction (N) away from the centre and weight (mg).

18.4 Conical Pendulum

A small block of mass m is rotated in a horizontal circle with the help of a string of length l connected to m. The other end of the string is fixed to a point O vertically above the cnetre of the circle so that the string is always inclined with the vertical at an angle θ. This arrangement is known as a conical pendulum.





From the force diagram of the block.

Along the vertical : $T\cos\theta = mg$ ...(i)

Net force towards centre : $T\sin\theta = ma$

$T\sin\theta = m\omega^2 r$ ...(ii)

From (i) and (ii), we have

$$\omega^2 = \frac{g\tan\theta}{r} = \frac{g\tan\theta}{\ell\sin\theta} = \frac{g}{\ell\cos\theta}$$

⇒ Time period $= T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$

Note...

- If h is the height of point O above the centre of the circle. then time period $= 2\pi\sqrt{h/g}$
- For a conical pendulum, $\omega^2 \ell\cos\theta = g$

⇒ $\omega > \sqrt{g/\ell}$ (Because $\cos\theta < 1$)

18.5 Motion in a Vertical Circle

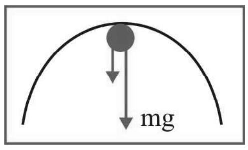
Example : A mass m is tied to a string of length l and is rotated in a vertical circle with centre at the other end of the string.

(a) Find the minimum velocity of the mass at the top of the circle so that it is able to complete the circle.

(b) Find the minimum velocity at the bottom of the circle.

At all positions, there are two forces acting on the mass : its own weight & the tension in the string.

Let the radius of the circle = l



(a) **<u>At the top</u>** : Let v_t = velocity at the top

net force towards centre $= \frac{mv_t^2}{\ell}$

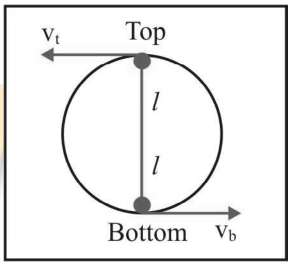
$$T + mg = \frac{mv_t^2}{\ell} \Rightarrow T = \frac{mv_t^2}{\ell} - mg$$

For the movement in the circle, the string should remain tight i.e. the tension must be positive at all positions.

As the tension is minimum at the top $T_{top} \geq 0$

⇒ $\frac{mv_t^2}{\ell} - mg \geq 0 \Rightarrow v_t \geq \sqrt{\ell g}$

⇒ minimum or critical velocity at the top $= \sqrt{1g}$



(b) Let V_b be the velocity at the bottom. As the particle goes up, its KE decreases and GPE increases.

⇒ loss in KE = gain in GPE

⇒ $\frac{1}{2}mv_b^2 - \frac{1}{2}mv_1^2 = mg(21)$

$$v_b^2 = v_t^2 + 4g\ell$$

$$(v_b)_{min} = \sqrt{(v_t^2)_{min} + 4g\ell} = \sqrt{5g\ell}$$

Note...

When a particle moves in a vertical circle, its speed decreases as it goes up and its speed increases as it comes down. Hence it is an example of non-uniform circular motion.