PROJECT 2

AIM

To determine the radius of gyration about the centre of mass of a metre scale used as a bar pendulum.

APPARATUS AND MATERIAL REQUIRED

A metre scale with holes at regular intervals, knife edge shaped axle, a rigid support, two glass plates (to be used for suspension plane), spring balance, spirit level, telescope fixed on a stand, stop-watch and graph paper.

PRINCIPLE

A rigid body oscillating in a vertical plane about a horizontal axis passing through it is known as compound pendulum. The point in the body through which the axis of rotation passes is known as centre of suspension.

The time period of a compound pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{mgl}} \qquad \text{(P 2.1)}$$

where m is the mass of the rigid body, l is the distance of the point of suspension from the centre of gravity, I is the moment of inertia of the body about the axis of oscillation and g is the acceleration due to gravity.

If K is the radius of gyration of the body about an axis through the centre of gravity, then the moment of inertia about the centre of suspension is

$$I = m\,(K^2 + l^2)$$

$$= m\ l\ \ l+\frac{K^2}{l} \qquad \text{(P 2.2)}$$

Hence $T = 2\pi\sqrt{\dfrac{ml\left(l + \dfrac{K^2}{l}\right)}{mgl}} = 2\pi\sqrt{\dfrac{\left(l + \dfrac{K^2}{l}\right)}{g}}$

(P 2.3) or $T = 2\pi\sqrt{\dfrac{L}{g}}$

(P 2.4) where $L = (l + K^2/l)$

Eq. (P 2.4) can be written as

(P 2.5) $l \, . \, L = (l^2 + K^2) \qquad l^2 - l\,L + K^2 = 0$

Eq. (P 2.5) is quadratic in l and therefore has two roots, say l_1 and l_2 then

$$l_1 + l_2 = L \quad \text{and} \quad l_1 \; l_2 = K^2$$

or $K = \sqrt{l_1 \; l_2}$

PROCEDURE

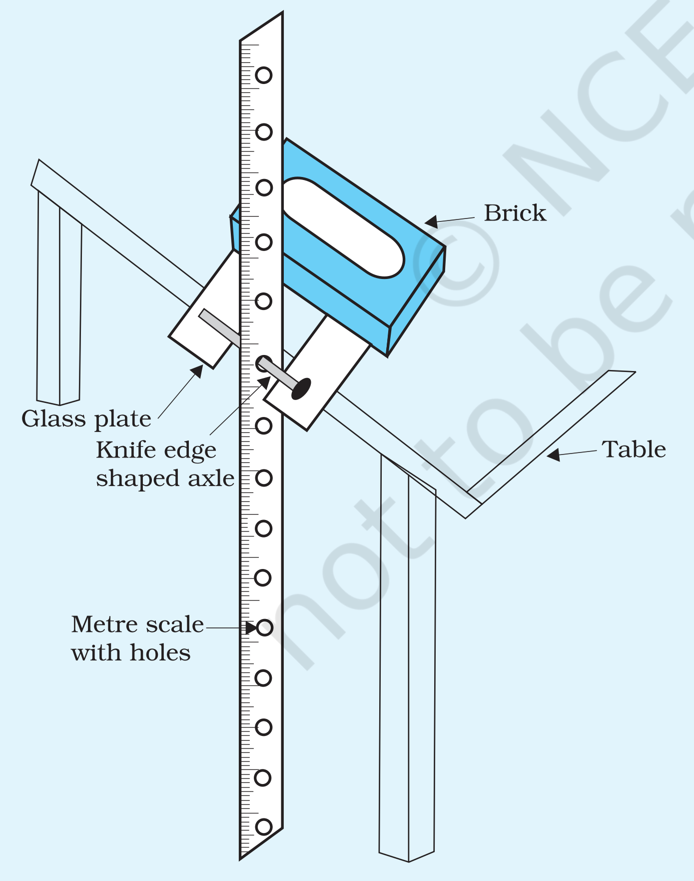

Fig. P 2.1: *A metre scale oscillating about a point close to C.G.*

1. Take a metre scale. Draw a line in the middle along its length. Drill holes of about 1.6 mm diameter on this line separated by a distance of 2 cm, starting from one end to the other.
2. Determine the centre of gravity of the scale by balancing it over a wedge.
3. Pass the knife edge shaped axle in the hole near one of the ends of the metre scale and let it rest on the suspension base having glass plates at its top.
4. Ensure that the glass plates fixed on the suspension plane be horizontal and in the same level so that when we suspend the metre scale by placing the knife edge we may be sure that the scale hangs vertically (Fig. P2.1).
5. Make a reference line, drawn on the paper strip, near the lower end of the pendulum and focus it with a telescope. Adjust the telescope until its vertical crosswire focuses on the reference line.

6. Displace the lower end of the scale horizontally through a small distance from its equilibrium position and then release it. The pendulum (metre scale) will begin to oscillate. Take care that the angular amplitude of oscillation is within 5° or 6° and pendulum oscillates in a vertical plane without any jerk.

7. Count zero when the reference mark on oscillating pendulum passes across the vertical crosswire of telescope and start the stop-watch at that instant (The counting of oscillations could be done visually, in case a telescope is not available).

8. Continue counting 2, 3, 4,... successively when the reference line progressively passes the vertical crosswire from the same side and note the time for 20 oscillations. Repeat the observations at least three times.

9. Measure from the lower end, the distance of the point of suspension.

10. Repeat Steps 7 and 9 after shifting the knife edge to the successive holes leaving two holes on either side of the centre of gravity of the pendulum. Take length of pendulum on one side of C. G. as positive while on the other side as negative. Record your observations in tabular form.

OBSERVATIONS

Table P. 2.1: Measurement of time period of compound pendulum

Hole No.	One side of C.G.					Hole No.	Other side of C.G.				
	Distance from C.G., l_1 (cm)	Time for 20 oscillations			Time period $T = \frac{t_1 + t_2 + t_3}{3}$ (s)		Distance from C.G., l_2 (cm)	Time for 20 oscillations			Time period $T' = \frac{t'_1 + t'_2 + t'_3}{3}$ (s)
		t_1	t_2	t_3				t'_1	t'_2	t'_3	

CALCULATION

1. Plot a graph between l and T by taking the l along x-axis and T along y-axis. The graph will consist of two symmetrical curves Fig. P 2.2. The point on the x-axis about which the graph is symmetrical is the centre of gravity of the metre scale pendulum.

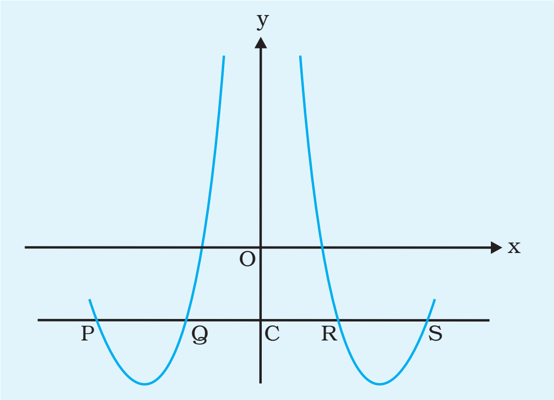

Fig. P 2.2: *Graph between distance from C.G. and time period*

2. Draw a line parallel to x-axis cutting the graph at points, *P*, *Q*, *R* and *S*

   (a) From the graph, CP= ... cm, CS= ... cm

   $$l_1 = \frac{CP + CS}{2} = \ldots \text{cm}$$

   (b) From the graph, CQ = ... cm, CR = ... cm

   $$l_2 = \frac{CQ + CR}{2} = \ldots \text{cm}$$

   (c) The radius of gyration $K = \sqrt{l_1 \; l_2}$

RESULT

The radius of gyration about the axis passing through the centre of mass of the metre scale is found as *K* = ... cm.

PRECAUTIONS

1. Pendulum should be hung vertically and knife edge be kept horizontal so that the pendulum oscillates in a vertical plane.
2. Note the time by the stop-watch leaving 5 or 6 initial oscillations so that effect of any irregularities in the oscillations get subsided.
3. Increase the number of observations for a given length of pendulum if time for 20 oscillations is to be measured without using a telescope.
4. Keep the fans off or else air droughts will shift the position of the scale and its oscillations will not remain in the same plane.

SOURCES OF ERROR

1. The metre scale may not have uniform mass distribution.
2. The wedge may not be sharp.
3. The holes drilled may not be colinear or have equally smooth inner surface.

DISCUSSION

1. If a metallic bar is used in place of wooden scale we would have better results as its inertia will hold it in position in a better way.

Moreover a metallic bar of homogeneous material and uniform cross-section can be easily made.

2. To draw smooth symmetrical graphs, we may make use of curved surface on the inside of set squares or by suitably bending plastic tongue cleaners or broomsticks.

SELF ASSESSMENT

1. How would you establish that the compound pendulum executes SHM?
2. By knowing the radius of gyration of the metre scale about its centre of mass, determine the moment of inertia of the same scale about an axis passing through the centre of mass.
3. Why do we get two L – T plots symmetrical about y-axis?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Increase the angular amplitude slowly and see how your result changes.
2. Note the angular amplitude at which the variation in your results is appreciable. How will you explain the changes?