PROJECT 1

AIM

To investigate whether the energy of a simple pendulum is conserved.

APPARATUS AND MATERIAL REQUIRED

A tall laboratory clamp stand with clamps, a split cork, a brick (or any heavy metallic weight) to be used as bob, strong cotton thread/ string (about 1.5 m to 2.0 m), stop-watch, ticker timer, paper tape, balance, wooden block, cellotape, metre scale and graph paper.

PRINCIPLE

Energy can neither be created nor destroyed, though it can be transformed from one form to another, and the sum of all forms of energies in the universe remains constant (Law of conservation of energy). In any isolated mechanical system with practically negligible/no dissipation of energy to overcome viscous drag/air resistance / friction, (as in case of a pendulum), the sum of the kinetic and potential energies remains constant. For small angular amplitude ($\theta \leq 15°$), the pendulum executes simple harmonic motion (SHM) with insignificant damping, i.e., loss of energy. Hence, an oscillating simple pendulum provides a convenient arrangement to investigate/ validate the law of conservation of energy for a mechanical system.

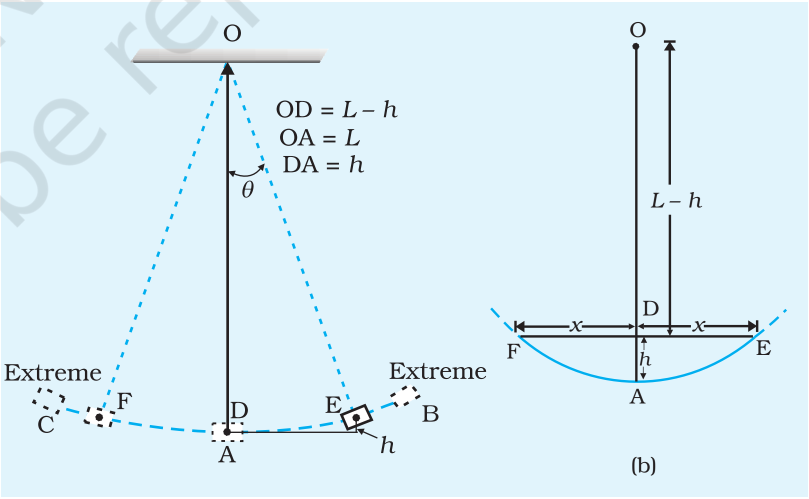

Fig. P 1.1: *An oscillating pendulum*

The oscillations of a simple pendulum of effective length L with mean position at point A and extreme positions at points B and C, are shown in Fig. P 1.1. In the extreme positions, i.e., at B and C the oscillating bob is raised to a certain height h (= AD) above the mean position where it possesses maximum potential energy but minimum kinetic energy. In the mean position, at A, it possesses maximum kinetic energy and minimum potential energy. At any intermediate position i.e., at E and F the bob will possess energy in the form of both kinetic and potential energies. The effective length L (= $l + r$) of the pendulum is taken from the point of suspension O to the centre of gravity of the bob (Fig P 1.1; also refer Experiment E 6). For small angular amplitudes (θ) (about 8° to 10°) the arc length EA = (FA) is about the same as linear distance ED = (FD) = x, the points E and F are symmetrically above point D.

From the geometry of the Fig. P 1.1, it follows

DF. DE = OD. DA

$x \times x = (L - h)\, h$

For small values of x and h (and $x << L$ and $h << x$)

(P 1.1) $$h = \frac{x^2}{L}$$

Then the potential energy of the bob (brick) of mass m at point E (or F)

(P 1.2) $$= mgh = \frac{mg}{L} x^2$$

The kinetic energy E possessed by the bob moving with velocity v at

(P 1.3) point E (or F) is = $\frac{1}{2} mv^2$

Then total energy of the bob is given by

(P 1.4) $$E = \frac{1}{2} mv^2 + \frac{mg}{L} x^2$$

Using this relation, now investigate whether the total energy E of the oscillating simple pendulum remains constant.

DEVICE FOR MEASURING SHORT TIME INTERVALS IN THE LABORATORY: TICKER TIMER

Ticker-timer is a device used for the measurement of short time-intervals in the laboratory. It can measure short time intervals of about 0.02s to much higher degree of accuracy as compared to that of a stop-watch (with least count of 0.1s). Ticker-timers are available in different designs.

A simple type of ticker-timer, as shown in Fig.P1.2, consists of a steel/metallic strip T which can be made to vibrate at a known frequency with the help of an electromagnet. The pointed hammer of the vibrating steel strip, T strikes a small carbon paper disc C under which a paper tape, is pulled by the oscillating object. The dot marks are marked on the paper tape by the pointed hammer when the strip vibrates.

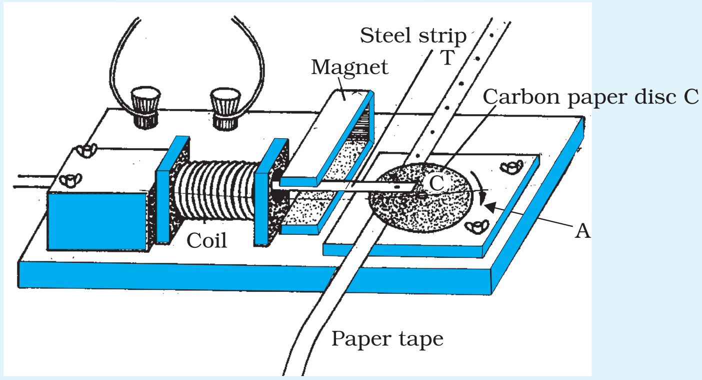

Fig. P1.2: *Ticker-timer*

The dot marks are obtained on the paper tape at regular (or equal) intervals of time. Each dot mark refers to a complete vibration of the vibrating steel strip. The time interval between the two consecutive dot marks can be taken as a unit of time for a tick. The time period of the vibrating strip is obtained from its given (known) frequency of vibration. When it is run on 6V step-down ac supply, its frequency is the same as that of ac mains (50 Hz, in India).

In this way, the measured time interval for one tick (between the two consecutive dot marks) can be converted into the basic unit, second, for time measurement. Thus, the ticker-timer can be used to measure accurately time interval of the order of 0.02 s in the laboratory.

PROCEDURE

1. Find the mass of the pendulum bob.
2. Determine r and l by metre scale. The length of the pendulum $L = l + r$.
3. Take the ticker-timer and place it at about the same level as the centre of the bob as shown in Fig. P 1.3. Fix the ticker-timer on a wooden block with tape, to ensure that its position is not disturbed when tape is pulled through it.
4. Attach the tip of the paper tape of the ticker-timer to the bob with the help of cellotape such that it is horizontal and lies in the plane in which centre of gravity of the bob lies in its rest position.

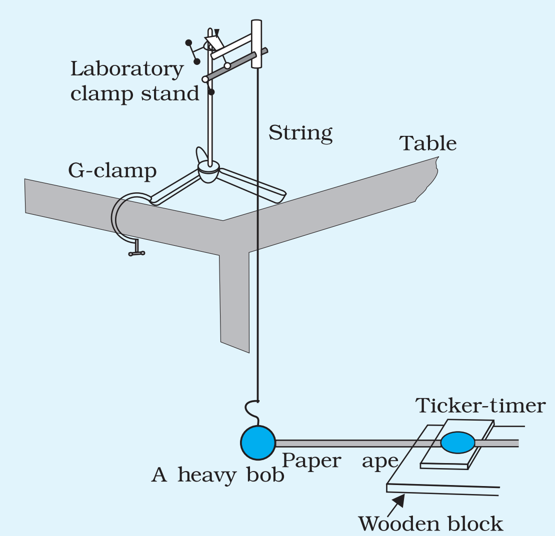

Fig. P 1.3: *Experimental setup for studying conservation of energy*

5. Pull the bob towards the timer such that its angular displacement ($\theta < 10^o$) is about one tenth of its length from the vertical position. Take care that the ticker tape is sufficiently light and is so adjusted that it easily moves by the pull of bob as soon as it begins to move.

6. Start the ticker-timer carefully and let the bob oscillate. While the bob moves towards the other side, it pulls the paper tape through the ticker-timer. Ticker timer, thus, records the positions of the bob at successive time intervals.

7. Switch off the ticker-timer when the brick reaches the other extreme end. Take out the paper tape and examine it. Extreme dot marks on the record of the tape represent the extreme positions B and C of the pendulum. The centre point A of this half oscillation is the centre of the two extreme dot marks, and may be marked by the half metre scale, as in Fig. P1.4.

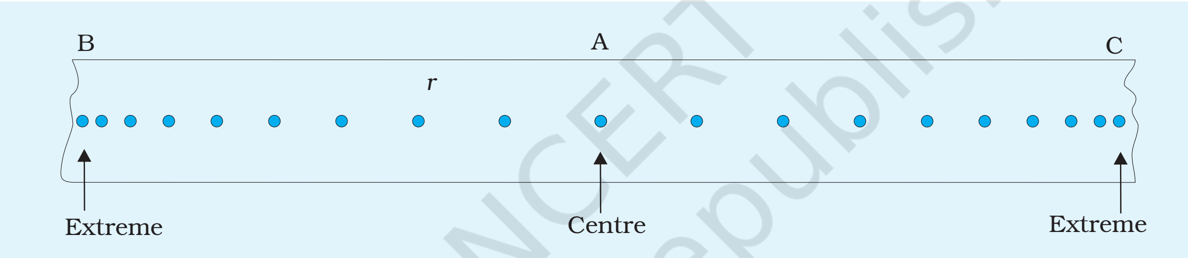

Fig. P 1.4: *Position of the oscillating bob marked on paper tape*

8. Measure the displacements of the bob corresponding to each dot (about 10 to 12) on either sides from the centre marked A as x_1, x_2, x_3, ... Find the time t_1, t_2, t_3, ... when each selected dot was made by counting the number of dots from the central point A, representing the mean position of the pendulum. If central point A is not coinciding with a dot marked by the ticker-timer, appropriate fraction of time-period of ticker-timer has to be added for finding correct t_1, t_2, t_3, ...

9. Record observations in the tabular form in SI units and proper significant figures.

10. Calculate the corresponding velocity for each selected position of the dot as v_i ($= \Delta x_i / \Delta t_i$). For this take one earlier and one later dot. The distance between these two dots is Δx_i and Δt_i is time to cover this distance. Then find magnitude of kinetic energy $\frac{1}{2}mv^2 = \frac{m}{2}\left(\frac{\Delta x_i}{\Delta t_i}\right)^2$ and potential energy

mgh_i [= $mg\,(x_i^2/L)$] of the pendulum bob. Find the sum of kinetic and potential energies in each case. Express the result in SI units and proper significant figures.

11. Plot a graph between the displacement (x_i) of the pendulum bob (distance of dots from the central dot) against the time.

12. Find the velocity (v) from the slope of the graph at five or six points on the left and also on the right of the mean position. Calculate the corresponding kinetic energy ($mv^2/2$) for each position of the points on the graph.

13. Plot another graph between kinetic energy and the position (x) of the bob. Find out the position of the point for which kinetic energy is minimum.

14. Calculate also the potential energy, PE $\left(= mg\dfrac{x_i^2}{L}\right)$, at the corresponding points at which you have calculated the kinetic energy. Plot the graph of potential energy (PE) against the displacement position (x) on the same graph on which you have plotted kinetic energy versus position graph.

15. Find the total mechanical energy E as the sum of kinetic energy and potential energy of the pendulum at each of the displacement positions x. Express the result in SI units with proper significant figures. Plot also a graph between the total mechanical energy E against displacement position (x) of the pendulum on the same graph on which you have plotted the graphs in Steps 13 and 14, i.e., for K.E. and P.E.

OBSERVATIONS

Measuring the mass of bob and effective length of simple pendulum

(a) Effective length of the simple pendulum

Least count of the metre scale = ... mm = ... cm

Length of the top of the brick from the point of suspension, l = ... cm = ... m

Diameter of the bob, $2r$ = ... cm

Effective length of the simple pendulum $L = (l + r)$ = ... cm = ... m

(b) Mass of the ... g

Time period (T) of ticker-timer = ... s

Fraction of T to be added for finding corrected T_i on left = ...

Fraction of T to be added for finding corrected T_i on right = ...

Table P1.1: Measuring the displacement and time using ticker-timer and the recorded tape

S. No.	S. No. of dot on tape (i)	Displacement (Distance of dot from centre, x_i) (cm)	Number of vibrations of ticker-timer between central and i[th] point	T_1 (s)	Velocity v (m s^{-1})
1	2nd left				
2	4th left				
3	6th left				

	2nd right				
	4th right				
	6th right				

(c) Plotting a graph between displacement and time

Take time t along x-axis and displacement x along y-axis, using the observed values from Table P1.1. Choose suitable scales on these axes to represent t and x. Plot a graph between t and x as shown in Fig. P1.5. What is the shape of x-t graph?

CALCULATION

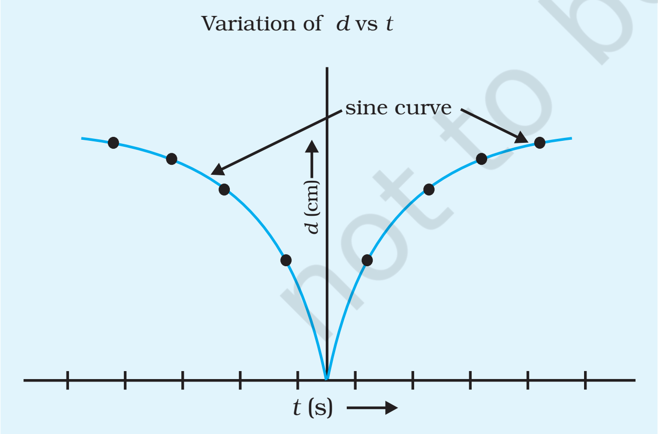

Fig. P1.5: *Graph between displacement and time of the oscillating bob*

(i) Find out from the graph (Fig. P1.5), the velocity of bob at five or six different points on the either side of the mean position O of the graph.

Compute the values of kinetic energy, using Eq. (P1.3), corresponding to each value of velocity obtained from the graph. Record these values in Table P1.2.

(ii) Plot a graph by taking displacement (distance) x along x-axis and kinetic energy (K.E.) along y-axis using the values from Table P1.2 as shown in Fig. P1.6.

(iii) Compute the values of potential energy using Eq. (P1.2), for each value of displacement in Step (ii) above.

Table P 1.2: Finding potential, kinetic and total energy of the oscillating bob

S. No.	Velocity, v (ms^{-1})	Kinetic Energy, $\frac{1}{2}mv^2$ (J)	Potential Energy, $mg\frac{x^2}{L}$ (J)	Total Energy = Potential Energy + Kinetic Energy (J)
1				
2				
3				
4				

(iv) Plot a graph by taking displacement (distance) x along x-axis and potential energy (P.E.) along y-axis on the same graph (Fig. P1.6).

(v) Compute the total energy E_T as the sum of the kinetic energy and potential energy at each of the displacement positions, x. Plot a graph by taking the displacement along x-axis and total energy E_T along y-axis on the same graph Fig. P1.6).

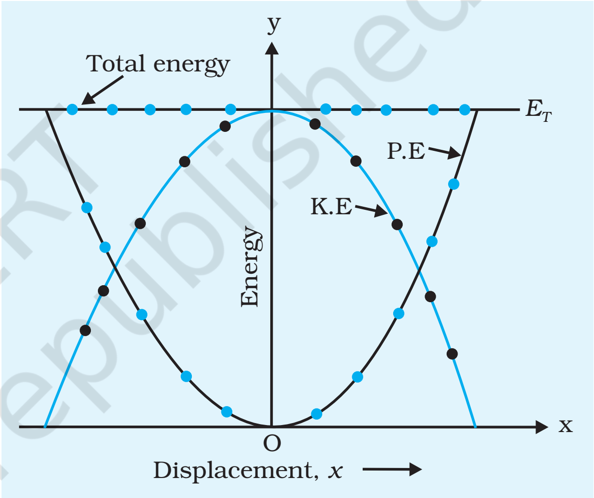

Fig. P 1.6: *Graph between displacement and energy of the oscillating bob*

RESULT

The total energy, as the sum of kinetic and potential energies, of the bob of the simple pendulum is conserved (remains the same) at all the points along its path.

DISCUSSION

1. Refer to points 3 to 5 under discussion given in Experiment E 6 on page 65.

2. Eq. P1.1 that expresses the relation between x, h and L for a simple pendulum, holds true under the conditions $h << x << L$ for small angular amplitudes ($\theta < 10°$) of the pendulum.

3. Linear displacement x of the bob, about (1/8)th to (1/10)th of the effective length of the pendulum corresponds to angular displacement (θ) of about 8° to 10° for small angular amplitudes,

the displacement (distance) of a dot mark on the paper tape from the central point/position truly represent corresponding displacement of the pendulum bob from its central (mean) position.

4. The shape of the graphs shown in Fig. P1.5 and Fig. P1.6 correspond to ideal conditions in which no energy is lost due to friction and air drag. The graph drawn on the basis of observed data may differ due to error in data collection and friction.

SELF ASSESSMENT

1. Identify the shape of displacement time graph, you have drawn for the oscillating simple pendulum. Interpret the graph.

2. Identify the shape of kinetic energy-displacement and potential energy-displacement graphs, you have drawn for the simple pendulum.

   Study the change in potential energy and kinetic energy at each of the displacement positions. Interpret these graphs and see how these compare.

3. What is the shape of the graph between the total (mechanical) energy and displacement you have drawn for the simple pendulum? Interpret the graph to show what it reveals?