ACTIVITY 7

AIM

To study dissipation of energy of a simple pendulum with time.

APPARATUS AND MATERIAL REQUIRED

A heavy metallic spherical ball with a hook; a rigid support; a long fine strong cotton thread (1.5 m to 2m); metre scale; weighing balance; sheet of paper; cotton; cellophane sheet.

PRINCIPLE

When a simple pendulum executes simple harmonic motion, the restoring force F is given by

(A 7.1) $$F(t) = -kx(t)$$

Where $x(t)$ is the displacement at time t and $k = mg/L$, the symbols k, m, g and L have been explained in Experiment E 6. The displacement is given by

(A 7.2) $$x(t) = A_0 \cos(\omega t - \theta)$$

where ω is the (angular) frequency and θ is a constant. A_0 is the maximum displacement in each oscillation, which is called the amplititude. The total energy of the pendulum is given as

(A 7.3) $$E_\circ = \frac{1}{2} k A_0^2$$

The total energy remains a constant in an ideal pendulum, because its amplitude remains constant.

But in a real pendulum, the amplitude never remains constant. It decreases with time due to several factors like air drag, some play at the point of suspension, imperfection in rigidity of the string and suspension, etc. Therefore, the amplitude of A_0 falls with time at each successive oscillation. The amplitude becomes a function of time and is given by

(A 7.4) $$A(t) = A_0 e^{-\lambda t/2}$$

where A_0 is the initial amplitude and λ is a contant which depends on damping and the mass of the bob. The total energy of the pendulum at time t is then given by

$$E(t) = \frac{1}{2} kA^2(t)$$

$$= E_0 \, e^{-\lambda t} \qquad \text{(A 7.5)}$$

Thus, the energy falls with time, because some of the energy is being lost to the surroundings.

The frequency of a damped oscillator does not depend much on the amplitude. Therefore, instead of measuring the time, we can also measure the number of oscillations n. At the end of n oscillations, $t = nT$, where T is the time period. Then Eq. (A 7.5) can be written in the form $E_n = E_0 \, e^{-\alpha n}$ (A 7.6)

where $\alpha = \lambda t$

and E_n is the energy of the oscillator at the end of n oscillations.

PROCEDURE

1. Find the mass of the pendulum bob.
2. Repeat Steps 1 to 5 of Experiment E 6.
3. Fix a metre scale just below the pendulum so that it is in the plane of oscillations of the pendulum, and such that the zero mark of the scale is just below the bob at rest.
4. When the pendulum oscillates, you have to observe the point on the scale above which the bob rises at its maximum displacement. In doing this, do not worry about millimetre marks. Take observations only upto 0.5 cm.
5. Pull the pendulum bob so that it is above the 15 cm mark. Thus, the initial amplitude will be A_0 = 15 cm at n = 0. Leave the bob gently so that it starts oscillating.
6. Keep counting the number of oscillations when the bob is at its maximum displacement on the same side.
7. Record the amplitude A_n at the end of n oscillations for n = 5, 10, 15, ..., that is at the end of every five oscillations. You may even note A_n after every ten oscillations.
8. Plot a graph of A_n^2 versus n and intepret the graph (Fig. A 7.1).
9. Stick a bit of cotton or a small strip of paper to the bob so as to increase the damping, and repeat the experiment.

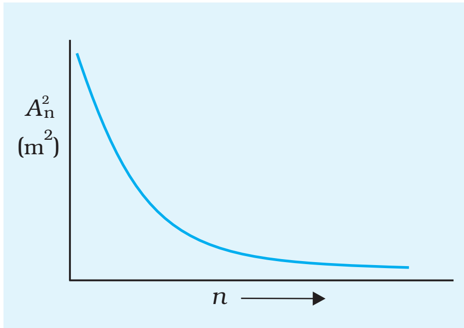

Fig. A 7.1: *Graph between A_n^2 and n for a simple pendulum*

OBSERVATIONS

Least count of the balance = ... g

Least count of the metre scale = ... cm

Mass of the pendulum bob. m = ... g

Radius (r) of the pendulum bob (given) = ... cm

Effective length of the pendulum (from the tip of the bob to the point of suspension), L = ... cm

Force constant, $k = mg/L$ = ... N m^{-1}

Initial amplitude of oscillation, A^0 = ... cm

Initial energy, $E_0 = 1/2\ (k\ A^2)$ = ... J

Table A 7.1 : Decay of amplitude with time and dissipation of energy of simple pendulum

S. No.	Amplitude, A_n (cm)	Number of oscillations, n	A_n^2	Energy of oscillater, E_n (J)	Dissipation of energy, (E_n-E_o) (J)
1					
2					
3					
4					

RESULT

From the graphs, we may conclude that the energy of a simple pendulum dissipates with time.

PRECAUTIONS

1. The experiment should be performed in a section of the laboratory where air flow is minimum.
2. The pendulum must swing for atleast a couple of oscillations before recording its amplitude, this will ensure that the pendulum is moving in the same plane.

SOURCES OF ERROR

1. Some movement of air is always there in the laboratory.
2. Accurate measurement of amplitude is difficult.

DISCUSSION

1. Which graph among the $A - n$ and $A^2 - n$ graph would you prefer for studying the dissipation of energy of simple pendulum with time and why?
2. How would the amplitude of oscillation change with time with the variation in (a) size and (b) mass of the pendulum bob; and (c) length of the pendulum?

SELF ASSESSMENT

1. Interpret the graph between A^2 and n you have drawn for a simple pendulum.
2. Examine how the amplitude of oscillations changes with time.
3. What does the decreasing amplitude of oscillation with time indicate in terms of variations in energy of simple pendulum with time.
4. In what way does graph between A and n differ from that between the A^2 and n graph, you have drawn.
5. Compare the $A^2 - n$ plots for
   (a) oscillations with small damping and
   (b) oscillations with large damping.

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

Take a plastic ball (5 cm diameter) make two holes in it along its diameter. Fill it with sand. Use the sand filled ball to make a pendulum of 100 cm length.

Swing the pendulum allowing the sand to drain out of the hole. Find the rate at which the amplitude of pendulum falls and compare it with the case when mass of the bob is constant.