

The Principle of Mathematical Induction

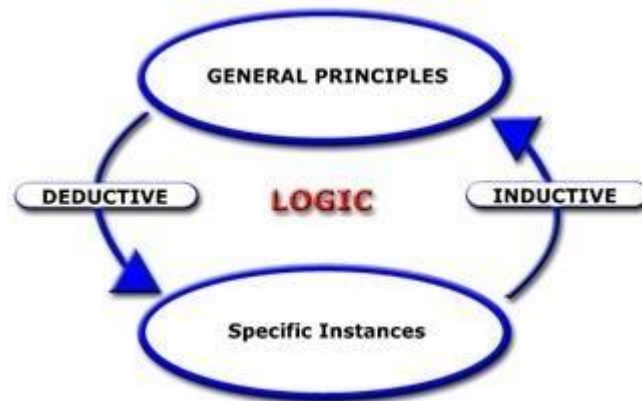
Deduction: Generalization of Specific Instance

Example : Rohit is a man & All men eat food, therefore, Rohit eats food.

Induction: Specific Instances to Generalization

Example : Rohit eats food. Vikash eats food. Rohit and Vikash are men. Then, All men eat food

Statement is true for $n=1$, $n=k$ & $n=k+1$, then, the Statement is true for all natural numbers n .



Steps of Principle of Mathematical Induction:

Step 1: Let $P(n)$ be a result or statement formulated in terms of n (given question).

Step 2: Prove that $P(1)$ is true

Step 3: Assume that $P(k)$ is true

Step 4: Using Step 3, prove that $P(k+1)$ is true

Step 5: Thus $P(1)$ is true and $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all natural numbers n .

Example: Prove that $2^n > n$ for all positive integers n

Solution:

Step 1: Let $P(n)$: $2^n > n$

Step 2: When $n = 1$, $2^1 > 1$. Hence $P(1)$ is true.

Step 3: Assume that $P(k)$ is true for any positive integer k , i.e., $2^k > k \dots (1)$

Step 4: We shall now prove that $P(k+1)$ is true whenever $P(k)$ is true.

Multiplying both sides of (1) by 2, we get

$$2 * 2^k > 2 * k$$

$$\text{i.e., } 2^{k+1} > 2k$$

$$\text{or, } 2^{k+1} > k + k$$

$$\text{or, } 2^{k+1} > k + 1 \quad (\text{since } k > 1)$$

Therefore, $P(k+1)$ is true when $P(k)$ is true. Hence, by principle of mathematical induction, $P(n)$ is true for every positive integer n .