Activity 8

Objective

To find the values of sine and cosine functions in second, third and fourth quadrants using their given values in first quadrant.

Material Required

Cardboard, white chart paper, ruler, coloured pens, adhesive, steel wires and needle.

Method of Construction

1. Take a cardboard of convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on chart paper.
3. Through the centre of the circle, draw two perpendicular lines $X'OX$ and YOY' representing x-axis and y-axis, respectively, as shown in Fig.8.1.

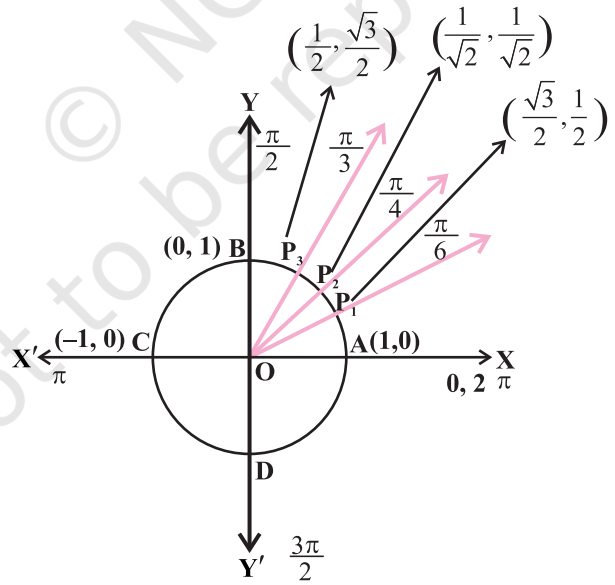

Fig. 8.1

4. Mark the points as A, B, C and D, where the circle cuts the *x*-axis and *y*-axis, respectively, as shown in Fig. 8.1.

5. Through O, draw angles P_1OX, P_2OX, and P_3OX of measures $\frac{\pi}{6}, \frac{\pi}{4}$ and $\frac{\pi}{3}$, respectively.

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle.

DEMONSTRATION

1. The coordinates of the point P_1 are $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$ because its *x*-coordinate is cos $\frac{\pi}{6}$ and *y*-coordinate is sin $\frac{\pi}{6}$. The coordinates of the points P_2 and P_3 are $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, respectively.

2. To find the value of sine or cosine of some angle in the second quadrant (say) $\frac{2\pi}{3}$, rotate the needle in anti clockwise direction making an angle P_4OX of measure $\frac{2\pi}{3} = 120°$ with the positive direction of *x*-axis.

3. Look at the position OP_4 of the needle in

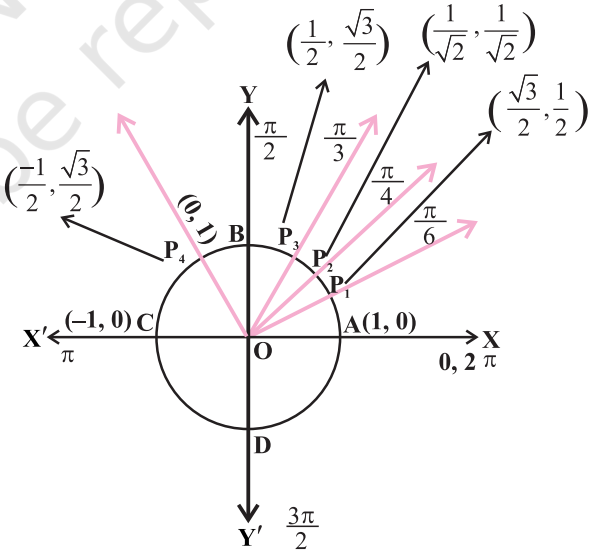

Fig. 8.2

Fig.8.2. Since $\frac{2\pi}{3}=\pi-\frac{\pi}{3}$, OP_4 is the mirror image of OP_3 with respect to y-axis. Therefore, the coordinate of P_4 are $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$. Thus $\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$ and $\cos\frac{2\pi}{3}=-\frac{1}{2}$.

4. To find the value of sine or cosine of some angle say, $\pi+\frac{\pi}{3}=\frac{4\pi}{3}$, i.e., $\frac{-2\pi}{3}$ (say) in the third quadrant, rotate the needle in anti clockwise direction making as an angle of $\frac{4\pi}{3}$ with the positive direction of x-axis.

5. Look at the new position OP_5 of the needle, which is shown in Fig. 8.3. Point P_5 is the mirror image of the point P_4 (since $\angle P_4OX' = P_5OX'$) with respect to x-axis. Therefore, co-ordinates of P_5 are $\left(-\frac{1}{2},\frac{-\sqrt{3}}{2}\right)$ and hence

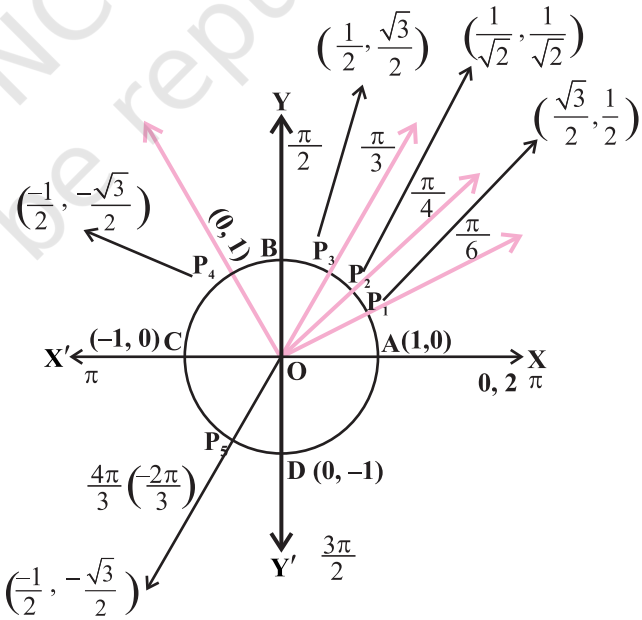

Fig. 8.3

$$\sin\left(-\frac{2\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \text{ and } \cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}.$$

6. To find the value of sine or cosine of some angle in the fourth quadrant, say $\frac{7\pi}{4}$, rotate the needle in anti clockwise direction making an angle of $\frac{7\pi}{4}$ with the positive direction of *x*-axis represented by OP_6, as shown in Fig. 8.4. Angle $\frac{7\pi}{4}$ in anti clockwise direction = Angle $-\frac{\pi}{4}$ in the clockwise direction.

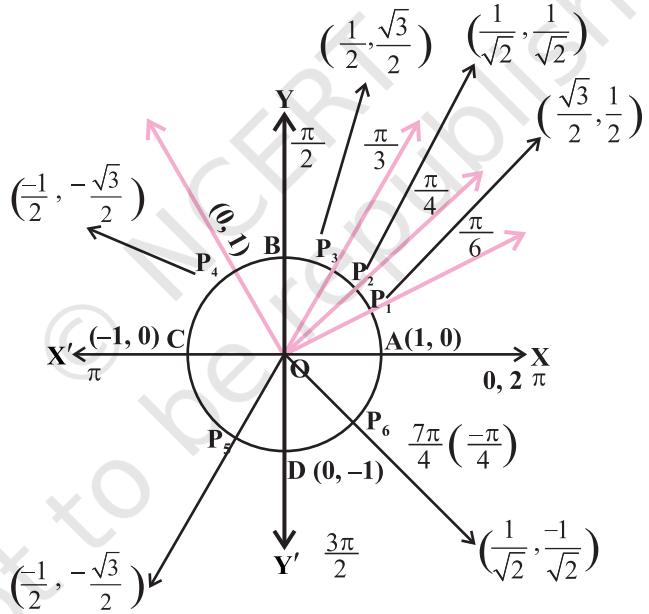

Fig. 8.4

From Fig. 8.4, P_6 is the mirror image of P_2 with respect to *x*-axis. Therefore, coordinates of P_6 are $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

Thus $\sin\left(\frac{7\pi}{4}\right)=\sin\left(-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

and $\cos\left(\frac{7\pi}{4}\right)=\cos\left(-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

8. To find the value of sine or cosine of some angle, which is greater than one revolution, say $\frac{13\pi}{6}$, rotate the needle in anti clockwise direction since $\frac{13\pi}{6}=2\pi+\frac{\pi}{6}$, the needle will reach at the position OP_1. Therefore, $\sin\left(\frac{13\pi}{6}\right)=\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$ and $\cos\left(\frac{13\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$.

Observation

1. Angle made by the needle in one complete revolution is _____.
2. $\cos\frac{\pi}{6}=$ _____ $=\cos\left(-\frac{\pi}{6}\right)$

   $\sin\frac{\pi}{6}=$_____ $=\sin(2\pi+$_____$)$.
3. sine function is non-negative in _____ and _____ quadrants.
4. cosine function is non-negative in _____ and _____ quadrants.

Application

1. The activity can be used to get the values for tan, cot, sec, and cosec functions also.
2. From this activity students may learn that
   $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = -\cos\theta$

   This activity can be applied to other trigonometric functions also.