Activity 29

Objective

Verification of the geometrical significance of derivative.

Material Required

Graph sheets, adhesive, hardboard, trigonometric tables, geometry box, wires.

Method of Construction

1. Paste three graph sheets on a hardboard and draw two mutually perpendicular lines representing x-axis and y-axis on each of them.

2. Sketch the graph of the curve (circle) $x^2 + y^2 = 25$ on one sheet.

3. On the other two sheets sketch the graphs of $(x-3)^2 + y^2 = 25$ and the curve $xy = 4$ (rectangular hyperbola).

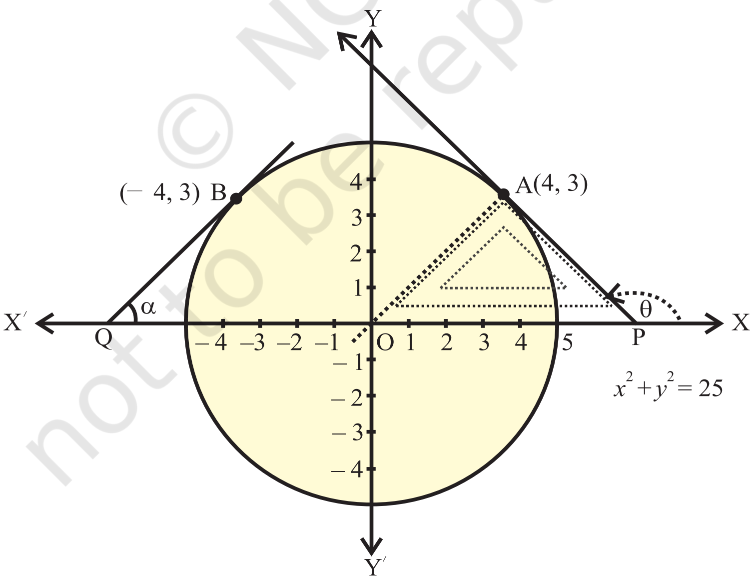

Fig. 29.1

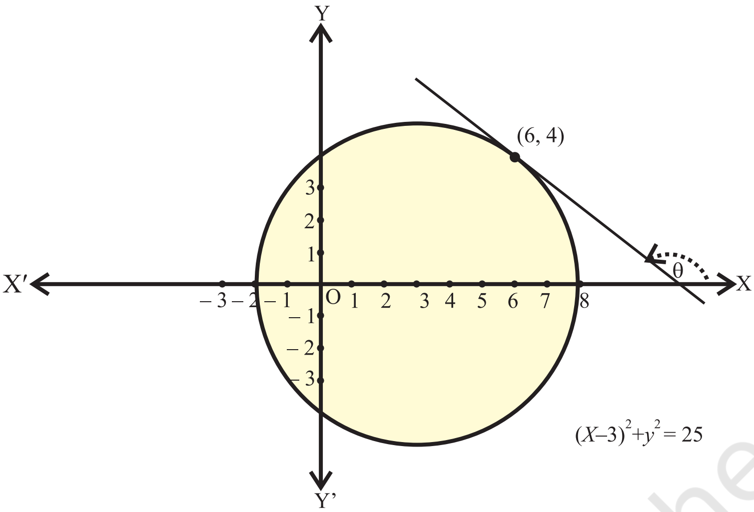

Fig 29.2

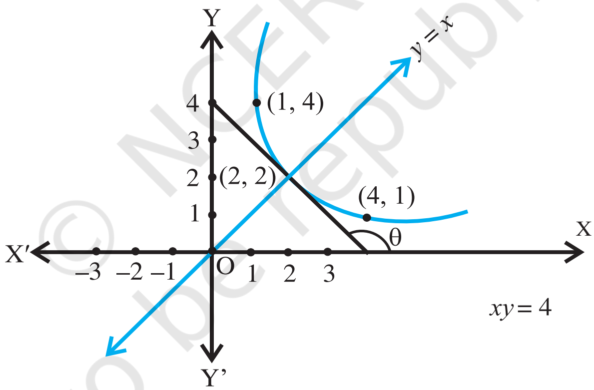

Fig. 29.3

DEMONSTRATION

1. Take first sheet on which, the graph of the circle $x^2 + y^2 = 25$ has been drawn (see Fig.29.1.
2. Take a point A (4, 3) on the circle.
3. With the help of a set square, place a wire in the direction OA and other perpendicular to OA at the point A to meet x-axis at a point (say P).

4. Measure the angle between the wire and the positive direction of x-axis at P (say θ).
5. Then find $\tan\theta$ (with the help of trigonometric tables)

   Now, $x^2 + y^2 = 25 \Rightarrow y = \sqrt{25 - x^2} \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{25 - x^2}}$.

   Find $\frac{dy}{dx}$ at the point (4, 3) and verify that $\left(\frac{dy}{dx}\right)$ at (4, 3) = $\tan\theta$.
6. Similarly, take another point (– 4, 3) on the circle. Verify that $\frac{dy}{dx}$ at (– 4, 3) = $\tan\alpha$ where α is the angle made by the tangent to the circle at the point (– 4, 3) with the positive direction of x-axis. (see Fig. 29.1).
7. Take other sheet with the graph of $(x - 3)^2 + y^2 = 25$ and take the point (6,4) on it and repeat the above process using set square and wires as shown in Fig. 29.2, i.e. verify that $\frac{dy}{dx}$ at (6, 4) = $\tan\theta$.
8. Now take the third sheet, showing the graph of the curve $xy = 4$. Take the point (2, 2) on it. Place one perpendicular side of set square along the line $y = x$ and a wire along the other side touching the curve at the point (2, 2) and find the angle made by the wire with the positive direction of x-axis as shown in Fig. 29.3. Let it be θ. Verify that $\frac{dy}{dx}$ at (2, 2) = $\tan\theta$.

OBSERVATION

1. For the curve $x^2 + y^2 = 25$, $\frac{dy}{dx}$ at the point (3, 4) = _________. Value of θ = ______ $\tan\theta$ = _________ $\frac{dy}{dx}$ at (3, 4) = __________.

2. For the curve $x^2 + y^2 = 25$, $\frac{dy}{dx}$ at $(-4, 3) =$ ________, $\tan \alpha =$ ________

, $\frac{dy}{dx}$ at $(-4, 3) =$ ________.

3. For the curve $(x-3)^2 + y^2 = 25$, $\frac{dy}{dx}$ at $(6, 4) =$ _____ , value of $\theta =$ ________,

$\tan \theta =$ _______, $\frac{dy}{dx}$ at $(6, 4) =$ ________.

4. For the curve $xy = 4$, $\left(\frac{dy}{dx}\right)$ at $(2, 2) =$ ______ ,

$\theta =$ ____, $\tan \theta =$ _______.

NOTE

The activity may be repeated by taking point (4, 3) on first sheet, (0, 4) on second sheet and (1, 4) on the third sheet.

APPLICATION

Same activity can be used to verify the result that the slope of the tangent at a point is equal to the value of the derivative at that point for other curves.