Activity 18

Objective

To demonstrate that the Arithmetic mean of two different positive numbers is always greater than the Geometric mean.

Material Required

Coloured chart paper, ruler, scale, sketch pens, cutter.

Method of Construction

1. From chart paper, cut off four rectangular pieces of dimension $a \times b$ ($a > b$).
2. Arrange the four rectangular pieces as shown in figure. 18.

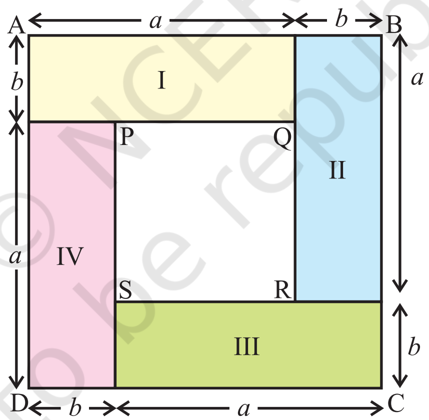

Fig. 18

Demonstration

1. ABCD is a square of side $(a + b)$ units.
2. Area ABCD $= (a + b)^2$ sq. units.
3. Area of four rectangular pieces $= 4\ (ab) = 4ab$ sq. units.

4. PQRS is a square of side $(a - b)$ units.
5. Area ABCD = Sum of the areas of four rectangular pieces + area of square PQRS.

$\therefore$ Area ABCD > sum of the areas of four rectangular pieces

i.e., $(a + b)^2 > 4ab$

or $\left(\frac{a+b}{2}\right)^2 > ab$

$\therefore \frac{a+b}{2} > \sqrt{ab}$, i.e., A.M. > G.M.

Observation

Take $a = 5$cm, $b = 3$cm

$\therefore$ AB = $a + b$ = ________ units.

Area of ABCD = $(a + b)^2$ = _______ sq. units.

Area of each rectangle = ab = _______ sq. units.

Area of square PQRS = $(a - b)^2$ = _________ sq. units.

Area ABCD = 4 (area of rectangular piece) + Area of square PQRS

_______ = 4 (_______) + (_______)

$\therefore$ ________ > 4 (_______)

i.e. $(a + b)^2 > 4ab$ or $\left(\frac{a+b}{2}\right)^2 > ab$

or $\frac{a+b}{2} > \sqrt{ab}$ $\therefore$ AM > GM