

Activity 17

OBJECTIVE

An alternative approach to obtain formula for the sum of squares of first n natural numbers.

MATERIAL REQUIRED

Wooden/plastic unit squares, coloured pencils/sketch pens, scale.

METHOD OF CONSTRUCTION

1. Take unit squares, 1, 4, 9, 16, 25 ... as shown in Fig. 17.1 and colour all of them with (say) Black colour.

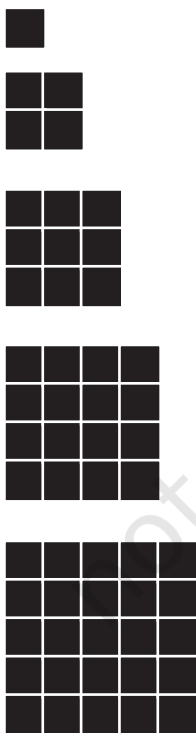


Fig. 17.1

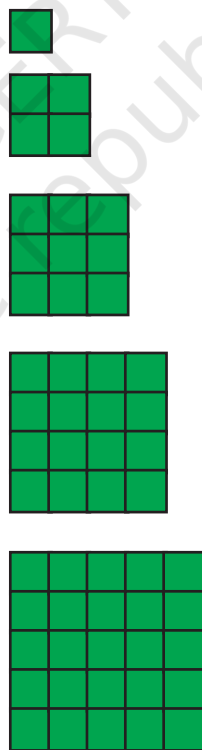


Fig. 17.2

- Take another set of unit squares 1, 4, 9, 16, 25 ... as shown in Fig. 17.2 and colour all of them with (say) green colour.
- Take a third set of unit squares 1, 4, 9, 16, 25 ... as shown in Fig. 17.3 and colour unit squares with different colours.
- Arrange these three set of unit squares as a rectangle as shown in Fig. 17.4.

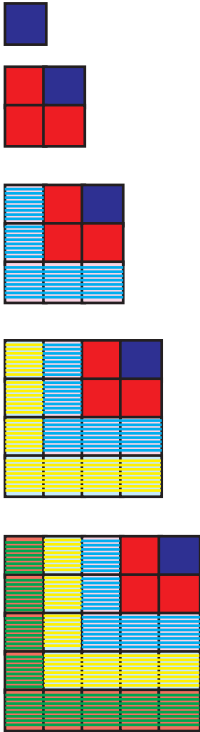


Fig. 17.3

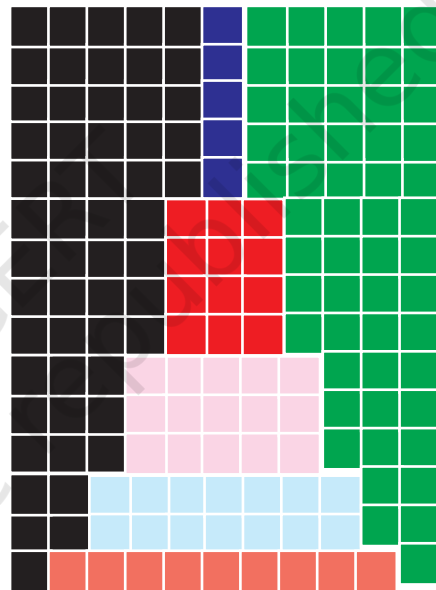


Fig. 17.4

DEMONSTRATION

- Area of one set as given in Fig. 17.1

$$= (1 + 4 + 9 + 16 + 25) \text{ sq. units}$$

$$= (1^2 + 2^2 + 3^2 + 4^2 + 5^2) \text{ sq. units.}$$

- Area of three such sets = $3 (1^2 + 2^2 + 3^2 + 4^2 + 5^2)$

$$3. \quad \text{Area of rectangle} = 11 \times 15 = [2(5) + 1] \left[\frac{5 \times 6}{2} \right]$$

$$\therefore 3(1^2 + 2^2 + 3^2 + 4^2 + 5^2) = \frac{1}{2} [5 \times 6] [2(5) + 1]$$

$$\text{or } 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{1}{6} [5 \times (5 + 1)] [2(5) + 1].$$

OBSERVATION

$$3(1^2 + 2^2 + 3^2 + 4^2 + 5^2) = \frac{1}{2} (\text{---} \times \text{---}) (\text{---} + 1)$$

$$\Rightarrow 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{1}{6} (\text{---} \times \text{---}) (\text{---} + 1)$$

$$\therefore 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = \frac{1}{6} (\text{---} \times \text{---}) (\text{---} + 1)$$

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + 10^2 = \frac{1}{6} (\text{---} \times \text{---}) (\text{---} + 1).$$

APPLICATION

This activity may be used to establish

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1).$$