Activity 16

OBJECTIVE

To obtain formula for the sum of squares of first n-natural numbers.

MATERIAL REQUIRED

Wooden/plastic unit cubes, coloured papers, adhesive and nails.

METHOD OF CONSTRUCTION

1. Take 1 (= 1^2) wooden/plastic unit cube Fig.16.1.
2. Take 4 (= 2^2) wooden/plastic unit cubes and form a cuboid as shown in Fig.16.2.
3. Take 9 (= 3^2) wooden/plastic unit cubes and form a cuboid as shown in Fig.16.3.
4. Take 16 (= 4^2) wooden/plastic unit cubes and form a cuboid as shown in Fig. 16.4 and so on.
5. Arrange all the cube and cuboids of Fig. 16.1 to 16.4 above so as to form an echelon type structure as shown in Fig.16.5.
6. Make six such echelon type structures, one is already shown in Fig. 16.5.
7. Arrange these five structures to form a bigger cuboidal block as shown in Fig. 16.6.

Fig 16.1

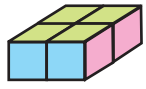

Fig 16.2

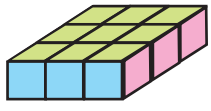

Fig 16.3

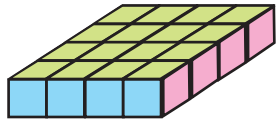

Fig 16.4

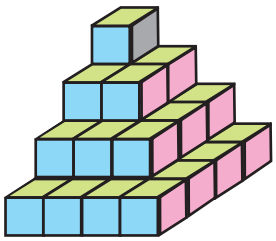

Fig. 16.5

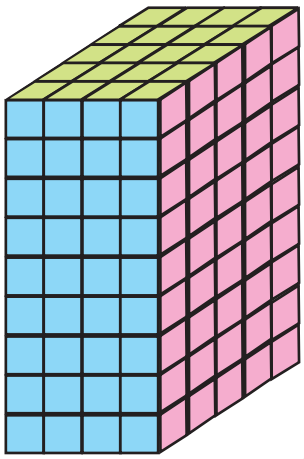

Fig. 16.6

Demonstration

1. Volume of the structure as given in Fig. 16.5

   $= (1 + 4 + 9 + 16)$ cubic units $= (1^2 + 2^2 + 3^2 + 4^2)$ cubic units.

2. Volume of 6 such structures $= 6\,(1^2 + 2^2 + 3^2 + 4^2)$ cubic units.

3. Volume of the cuboidal block formed in Fig. 16.6 (which is cuboid of dimensions $= 4 \times 5 \times 9) = 4 \times (4 + 1) \times (2 \times 4 + 1)$.

4. Thus, $6\,(1^2 + 2^2 + 3^2 + 4^2) = 4 \times (4 + 1) \times (2 \times 4 + 1)$

   i.e., $$1^2 + 2^2 + 3^2 + 4^2 = \frac{1}{6}[4 \times (4+1) \times (2 \times 4+1)]$$

Observation

1. $1^2 + 2^2 + 3^2 + 4^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

2. $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

3. $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 10^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

4. $1^2 + 2^2 + 3^2 + 4^2 \ldots + 25^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

5. $1^2 + 2^2 + 3^2 + 4^2 \ldots + 100^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

APPLICATION

This activity may be used to obtain the sum of squares of first n natural numbers as $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6} n\,(n + 1)\,(2n + 1)$.