Activity 11

Objective

To inerpret geometrically the meaning of $i=\sqrt{-1}$ and its integral powers.

Material Required

Cardboard, chart paper, sketch pen, ruler, compasses, adhesive, nails, thread.

Method of Construction

1. Paste a chart paper on the cardboard of a convenient size.
2. Draw two mutually perpendicular lines $X'X$ and $Y'Y$ interesting at the point O (see Fig. 11).
3. Take a thread of a unit length representing the number 1 along OX . Fix one end of the thread to the nail at 0 and the other end at A as shown in the figure.
4. Set free the other end of the thread at A and rotate the thread through angles of 90º, 180º, 270º and 360º and mark the free end of the thread in different cases as A_1, A_2, A_3 and A_4 , respectively, as shown in the figure.

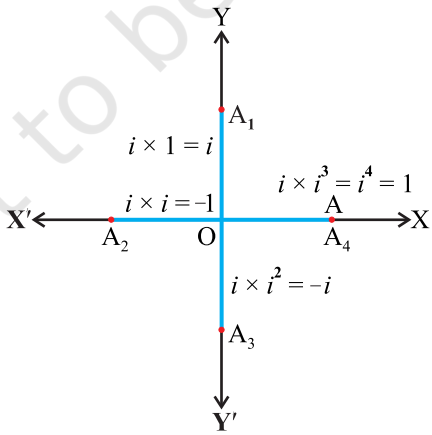

Fig. 11

Demonstration

1. In the argand plane, OA, OA_1, OA_2, OA_3, OA_4 represent, respectively, 1, i, –1, – i, 1.
2. $OA_1 = i = 1 \times i$, $OA_2 = -1 = i \times i = i^2$, $OA_3 = -i = i \times i \times i = i^3$ and so on. Each time, rotation of OA by 90º is equivalent to multiplication by i. Thus, i is referred to as the multiplying factor for a rotation of 90º.

Observation

1. On rotating OA through 90º, $OA_1 = 1 \times i =$ _____.
2. On rotating OA through an angle of 180°, $OA_2 = 1 \times$ __ $\times$ __ = _____.
3. On rotation of OA through 270º (3 right angles), $OA_3 =$

   1 $\times$ _____ $\times$ _____$\times$ _____ = _____.
4. On rotating OA through 360º (4 right angles),

   $OA_4 = 1 \times$ _____ $\times$ _____$\times$ _____ $\times$ _____ = _____.
5. On rotating OA through n-right angles

   $OA_n = 1 \times$ _____ $\times$ _____$\times$ _____ $\times$ _____ $\times$ ... n times = _____.

Application

This activity may be used to evaluate any integral power of i.